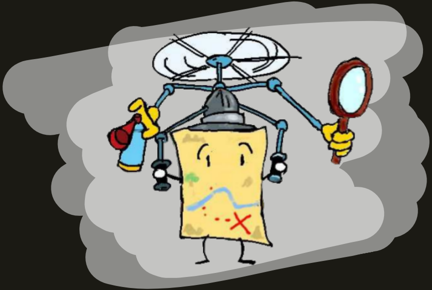
ANALYSIS OF PARAMETERS FOR LARGE COMBINATORIAL MAPS

Julien COURTIEL (AMACC, Caen)



WORK IN PROGRESS ?!!

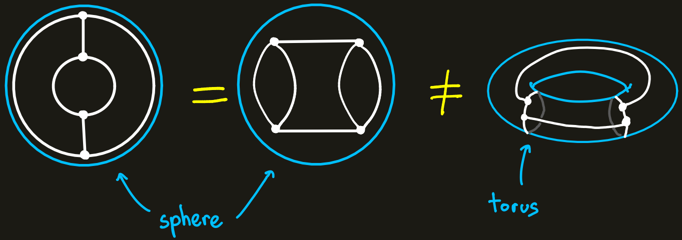
Séminaire ALGO Caen, 5 déc. 2017

Co-authors: Olivier BODINI (Paris 13), Sergey DOVGAL (Paris 13), Hsien-Kuei HWANG (Taïwan)

DEFINITION

combinatorial map = cellular embedding of a connected graph onto an oriented surface.

Examples:



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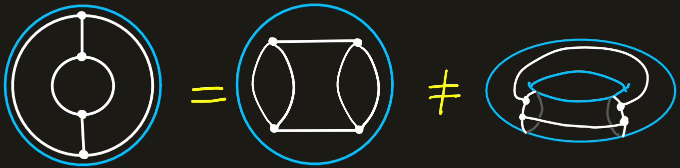

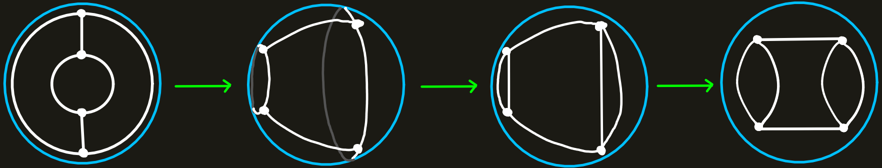

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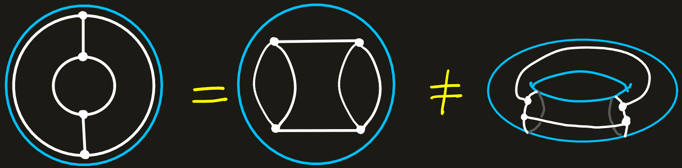
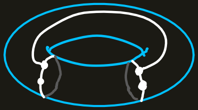


Counter-example:



not cellular because

is not a disk

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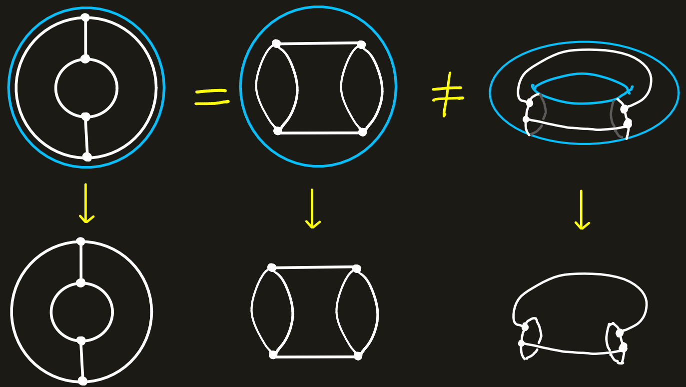

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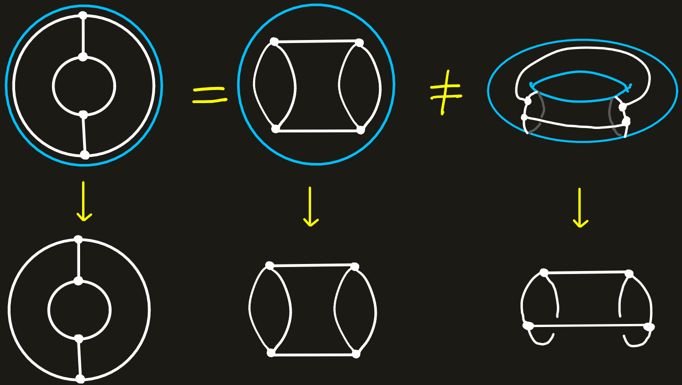

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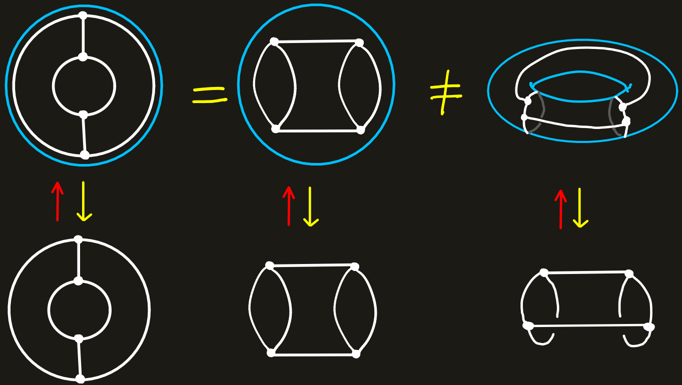
actually reversible



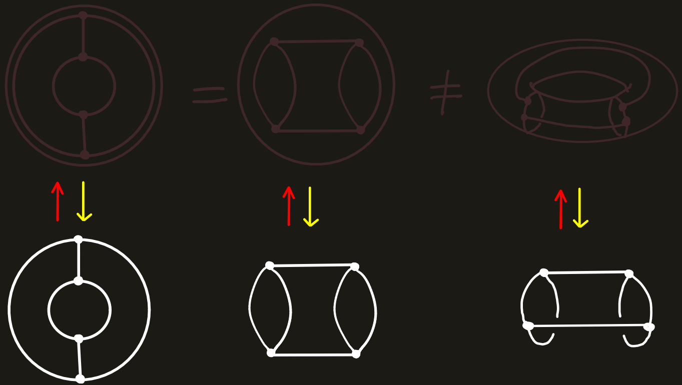
DEFINITION

FORGET IT!

combinatorial map = cellular embedding of a connected graph onto an oriented surface.

Examples:

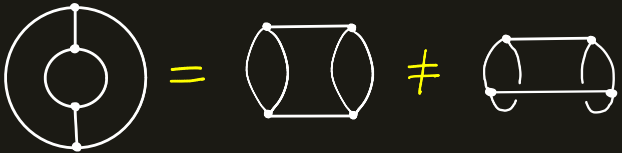
actually reversible



DEFINITION

combinatorial map = connected graph where we have cyclically ordered the half-edges around each vertex.

Examples:



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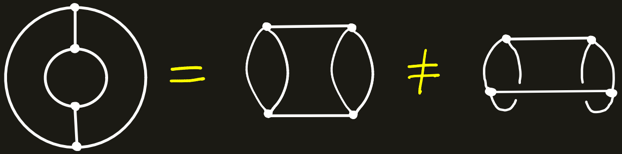
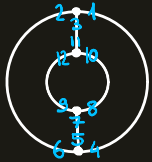
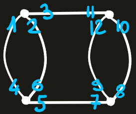


Why is  the same as  ?

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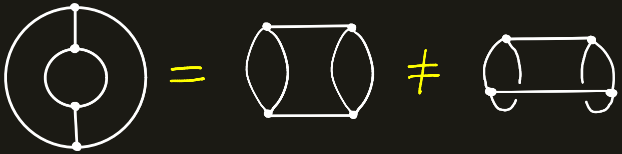
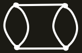
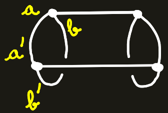


Why is  different from  ?



Absent pattern in  :

$a \leftrightarrow a'$ $a \circlearrowleft b$

$b \leftrightarrow b'$ $a' \circlearrowright b'$

DEFINITION

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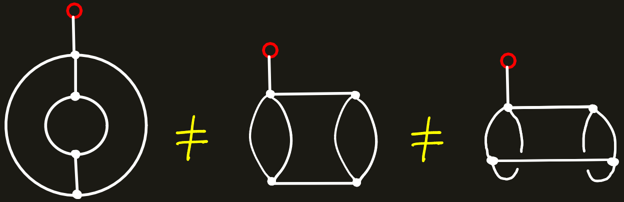
<u>Examples</u>:



We root every map on a leaf.

DEFINITION

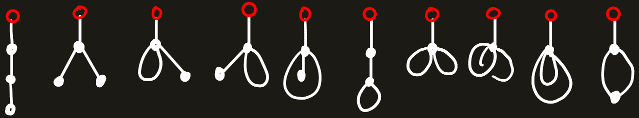
combinatorial map = connected graph where we have cyclically ordered the half-edges around each vertex.

1 edge 

1

2 edges 

2

3 edges 

10

RECURRENCE FORMULA

c_n = number of combinatorial maps with n edges

Recurrence formula: [Arquès Béraud]

$$c_1 = 1 \qquad c_n = \sum_{k=1}^{n-1} c_k c_{n-k} + (2n-3)\, c_{n-1}$$

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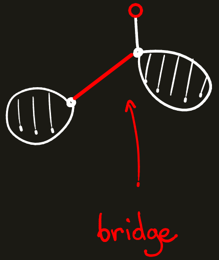
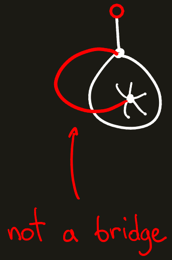
map =  or 

or 

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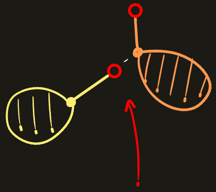
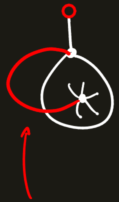
map =  or  or 

bridge

not a bridge

RECURRENCE FORMULA

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<u>Recurrence formula</u> : [Arquès Béraud]

number of possible insertions

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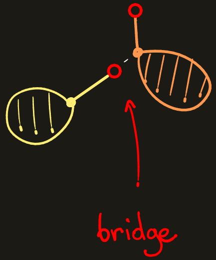
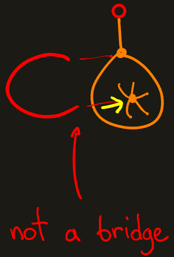
map =  or 

or 

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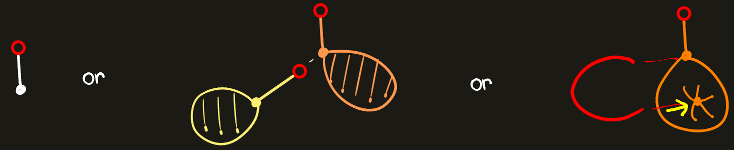

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$$C(z) = z + C(z)^2 + z\left(2z \frac{\partial C(z)}{\partial z} - C(z)\right)$$

WHY COUNTING MAPS WITH NO CONSIDERATION FOR GENUS?

→ Good framework to study parametric Riccati equations.

→ Connections with other combinatorial families.

- indecomposable chord diagrams (link with the Quantum Fields Theory)
- lambda-terms

WHY COUNTING MAPS WITH NO CONSIDERATION FOR GENUS?

→ Good framework to study parametric Riccati equations.

Part 2

→ Connections with other combinatorial families.

- indecomposable chord diagrams

  (link with the Quantum Fields Theory)

- lambda-terms

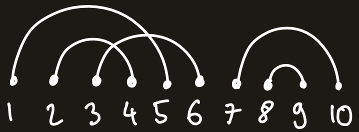
Part 1

PART I

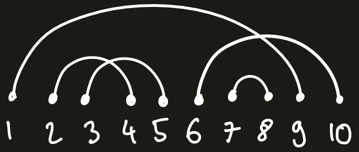
Connections with other combinatorial families

CHORD DIAGRAMS

diagram of n chords = matching of the set $\{1, \ldots, 2n\}$



indecomposable diagram = diagram that is <u>not</u> the concatenation of two diagrams.



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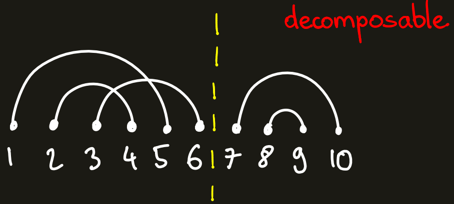

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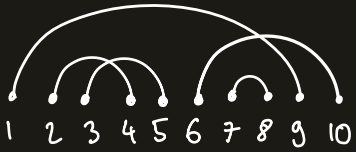

CHORD DIAGRAMS

1 chord

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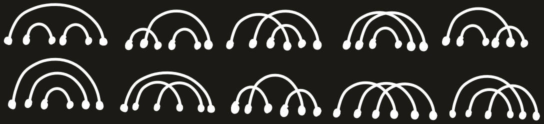
10



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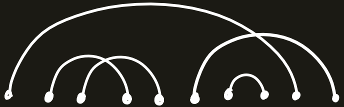


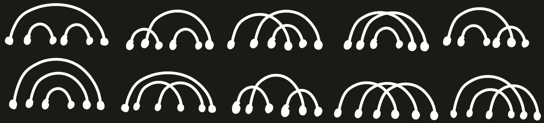
1 2 3 4 5 6 7 8 9 10

CHORD DIAGRAMS

1 chord  1

2 chords 2

3 chords 10



Proposition [Cvitanović Lautrup Pearson, Ossana de Mendez Rosenstiehl, Cori]

number of combinatorial maps with n edges

=

number of indecomposable diagrams with n chords

RECURRENCE FORMULA : THE COMEBACK

c_n = number of indecomposable diagrams with n chords

Recurrence formula :

$$c_1 = 1 \qquad c_n = \sum_{k=1}^{n-1} c_k c_{n-k} + (2n-3)\, c_{n-1}$$

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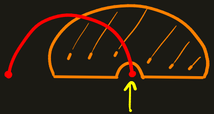
indecomposable diagram =  ou  ou 

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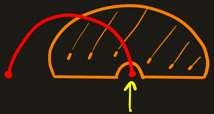
indecomposable diagram =  ou  ou 

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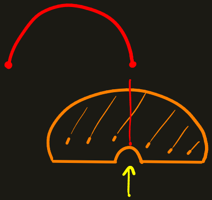
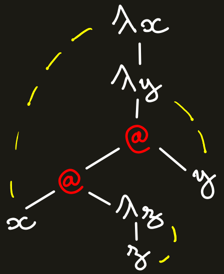
indecomposable diagram =  ou  ou 

LINEAR LAMBDA-TERMS



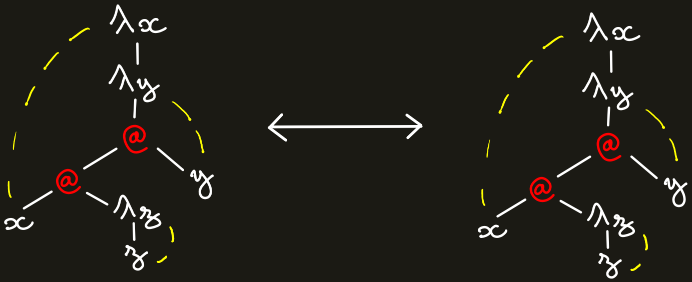
$\lambda x . \lambda y \, ((x \;\; \lambda z . z) \; y)$

linear lambda-term = Motzkin tree where each leaf is bound by a unary vertex and each vertex binds exactly one leaf.

Theorem [Bodini Gardy Gittenberg Jacquot]

linear lambda-terms $\longleftrightarrow$ trivalent maps

LINEAR LAMBDA-TERMS



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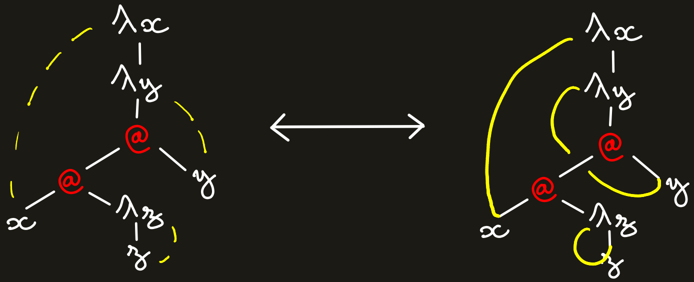

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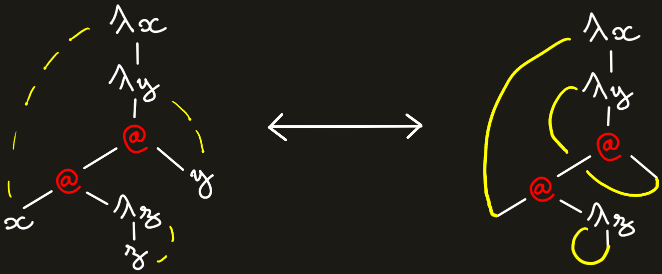

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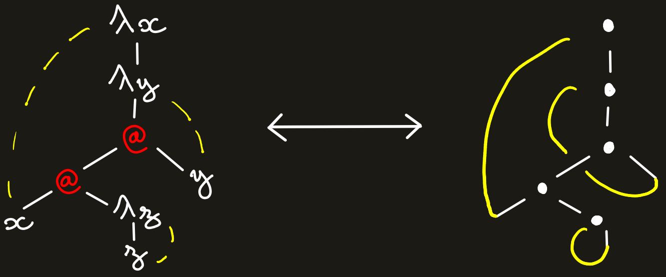

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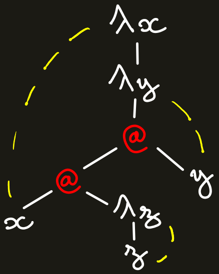

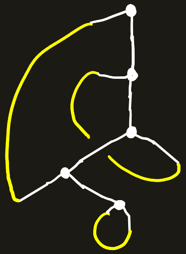

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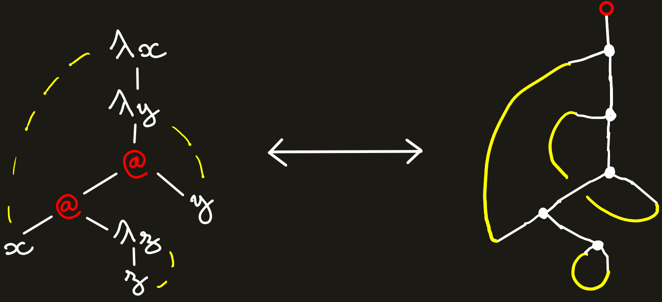

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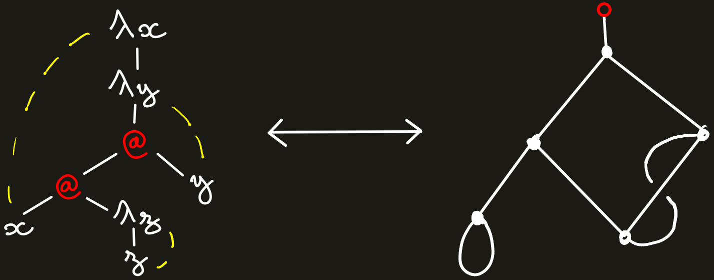

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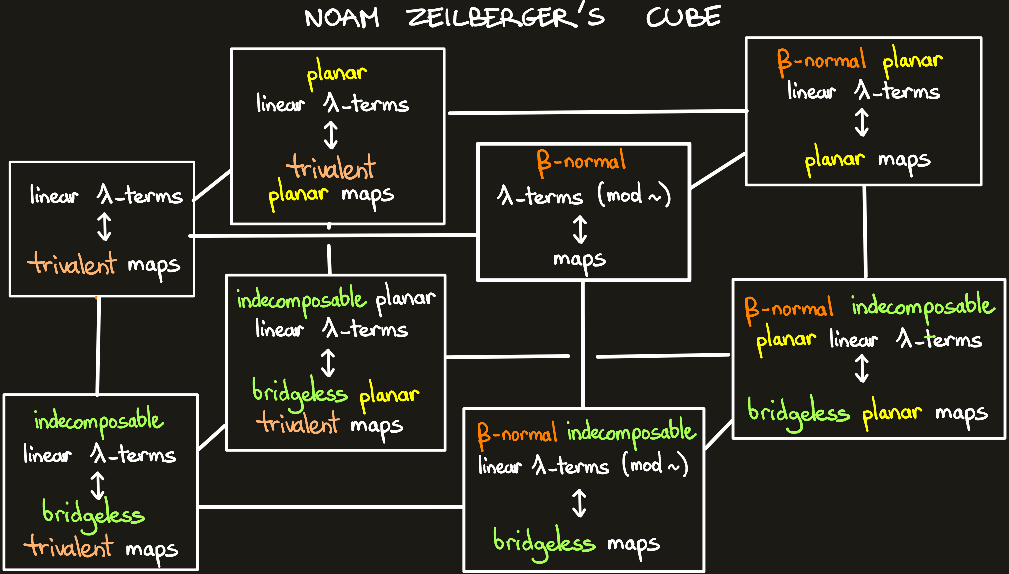
NOAM ZEILBERGER'S CUBE
planar linear λ-terms ↕ trivalent planar maps
β-normal planar linear λ-terms ↕ planar maps
linear λ-terms ↕ trivalent maps
β-normal λ-terms (mod ~) ↕ maps
indecomposable planar linear λ-terms ↕ bridgeless planar trivalent maps
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PART II

Asymptotic analysis of statistics on maps

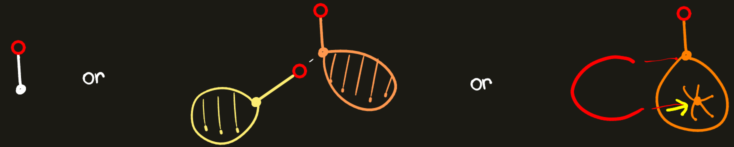
ASYMPTOTIC NUMBER OF MAPS

c_n = number of combinatorial maps with n edges

Recurrence formula :

$c_1 = 1$ $\qquad$ $c_n = \sum_{k=1}^{n-1} c_k c_{n-k} + (2n-3)\, c_{n-1}$

map =



Question 0 : Asymptotic estimate of c_n ?

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$$C(z) = z + C(z)^2 + z\left(2z \frac{\partial C(z)}{\partial z} - C(z)\right)$$

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<u>Idea</u>: (Formally) solve it !

ASYMPTOTIC NUMBER OF MAPS

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Riccati :(

$$C(z) = z + 2z^2\frac{\phi'(z)}{\phi(z)}$$

MAGIC TRICK!

linear :)

$$2z^2\phi''(z) + (5z-1)\phi'(z) + \phi(z) = 0$$

ASYMPTOTIC NUMBER OF MAPS

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$$2z^2 \phi''(z) + (5z-1)\phi'(z) + \phi(z) = 0$$

Solution: $\phi(z) = \sum_{n \geq 0} (2n-1)!!\, z^n$

$(2n-1)!! = (2n-1) \times (2n-3) \times \ldots \times 1$

ASYMPTOTIC NUMBER OF MAPS

$$C(z) = z + 2z^2 \frac{\phi'(z)}{\phi(z)}$$

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$(2n-1)!! = (2n-1) \times (2n-3) \times \ldots \times 1$

ASYMPTOTIC NUMBER OF MAPS

$$C(z) = z + 2z^2 \frac{\phi'(z)}{\phi(z)} \iff c_{n+1} = 2n\,\phi_n - \sum_{k=1}^{n-1} c_n\,\phi_{n-k}$$

$$2z^2 \phi''(z) + (5z-1)\,\phi'(z) + \phi(z) = 0$$

<u>Solution</u>: $$\phi(z) = \sum_{n\geq 0} (2n-1)!!\, z^n$$

$(2n-1)!! = (2n-1)\times(2n-3)\times \dots \times 1$

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$$C(z) = z + 2z^2 \frac{\phi'(z)}{\phi(z)} \iff c_{n+1} = 2n\phi_n - \sum_{k=1}^{n-1} c_k \phi_{n-k}$$

By some bootstrapping, $\boxed{c_n \sim \phi_n \left(2n - 1 - \frac{3}{2} n^{-1} - \frac{19}{4} n^{-2} + O(n^{-3})\right)}$

$$2z^2 \phi''(z) + (5z-1)\phi'(z) + \phi(z) = 0$$

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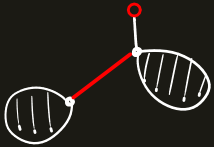
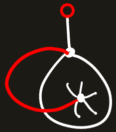
NUMBER OF VERTICES

$C(z) =$ generating function of maps where z counts the edges

<u>Equation</u>

$$C = z + C^2 + 2z^2\frac{\partial C}{\partial z} - zC$$

map =  or  or 

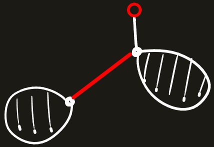
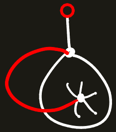
<u>Question</u> 1: behaviour of the number of vertices ?

NUMBER OF VERTICES

$C(z,u) =$ generating function of maps where z counts the edges and u counts the vertices

Equation

$$C = z u + C^2 + 2z^2 \frac{\partial C}{\partial z} - zC$$

map =  or  or 

Question 1: behaviour of the number of vertices ?

NUMBER OF VERTICES

$$C = z\,\mathcal{U} + C^2 + 2z^2\frac{\partial C}{\partial z} - zC$$

NUMBER OF VERTICES

$$C = z u + C^2 + 2z^2 \frac{\partial C}{\partial z} - z C$$

MAGIC TRICK!

$$C(z,u) = z u + 2z^2 \frac{\phi'(z,u)}{\phi(z,u)}$$

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<u>Solution</u>: $\phi(z,u) = 1 + \frac{u(u+1)}{2} z + \frac{u(u+1)(u+2)(u+3)}{2^2 \times 2!} z^2 + \dots + \frac{u(u+1)\dots(u+2n-1)}{2^n \times n!} z^n + \dots$

NUMBER OF VERTICES

Fact: $\phi(z,u)$ behaves like $C(z,u)$

Theorem:

For the uniform distribution of combinatorial maps,

$$\text{Number of vertices} \xrightarrow[\text{law}]{} \text{Gaussian law} \quad \begin{array}{l} \text{mean} \sim \ln(n) + \gamma + \dots \\ \text{variance} \sim \ln(n) + \gamma - \frac{\pi^2}{12} + \dots \end{array}$$

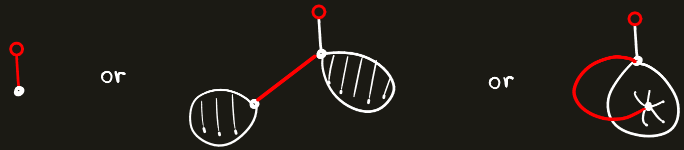
$$\phi(z,u) = 1 + \frac{u(u+1)}{2} z + \frac{u(u+1)(u+2)(u+3)}{2^2 \times 2!} z^2 + \dots + \frac{u(u+1)\dots(u+2n-1)}{2^n \times n!} z^n + \dots$$

NUMBER OF EDGES INCIDENT TO THE ROOT

$C(z,u)$ = generating function of maps where z counts the edges and u counts the number of edges incident to the root vertex.

Equation:

$$C(z,u) = zu + u\,C(z,u)\,C(z,1) + u\left(2z^2\frac{\partial C}{\partial z} - zC\right)$$

map = 

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MAGIC TRICK!

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$$2uz^2\,C'(z,u)\,\phi(z,1) + 2uz^2\,C(z,u)\,\phi'(z,1) = (1-2uz)\,C(z,u)\,\phi(z,1) - \phi(z,1)$$

NUMBER OF EDGES INCIDENT TO THE ROOT

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MAGIC TRICK!

$$C(z,1) = z + 2z^2\frac{\phi'(z,1)}{\phi(z,1)}$$

$$2uz^2 C'(z,u)\phi(z,1) + 2uz^2 C(z,u)\phi'(z,1) = (1-2uz)C(z,u)\phi(z,1) - \phi(z,1)$$

$$P(z,u) = \phi(z,u)\phi(z,1)$$

$$2uz^2 P'(z,u) = (1-2uz)\,P(z,u) - \phi(z,1)$$

almost linear!

NUMBER OF EDGES INCIDENT TO THE ROOT

$$C(z,u) = zu + u\,C(z,u)\,C(z,1) + u\left(2z^2\frac{\partial C}{\partial z} - zC\right)$$

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For the uniform distribution of combinatorial maps,

Number of edges incident to the root $\xrightarrow[\text{law}]{}$

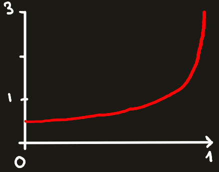
NUMBER OF EDGES INCIDENT TO THE ROOT

$$C(z,u) = zu + u\,C(z,u)\,C(z,1) + u\left(2z^2\frac{\partial C}{\partial z} - zC\right)$$

Theorem :

For the uniform distribution of combinatorial maps,

Number of edges incident to the root divided by n $\xrightarrow{\text{law}}$ Beta-law density $\frac{1}{2}(1-t)^{-\frac{1}{2}}$ sur $[0,1)$



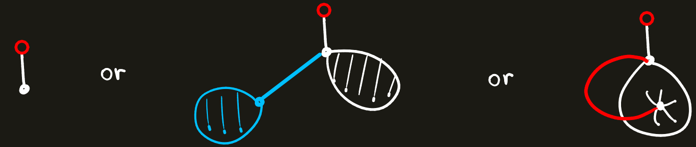
NUMBER OF COMPONENTS ATTACHED TO THE ROOT

$C(z,u)$ = generating function of maps where z counts the edges and u counts the number of connected components attached to the root vertex.

Equation:

$$C(z,u) = z + u\,C(z,u)\,C(z,1) + \left(2z^2\frac{\partial C}{\partial z} - zC\right)$$

map =



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Theorem:

$$\boxed{\text{Number of connected components attached to the root vertex} \xrightarrow[\text{law}]{} \text{Geometric law of parameter } 1/2.}$$

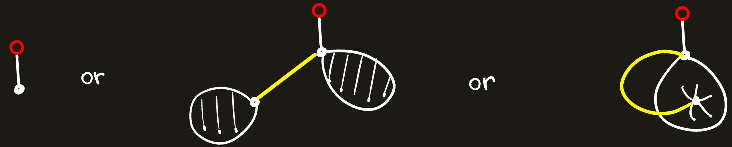
ROOT VERTEX DEGREE

$C(z,u)$ = generating function of maps where z counts the edges and u counts the degree of the root vertex

Equation:

$$C(z,u) = zu + u\,C(z,u)\,C(z,1) + u\left(2z^2\frac{\partial C}{\partial z} - zC\right) + (u^2-u)\frac{\partial C}{\partial u}$$

map =



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Theorem:

Degree of the root vertex $\xrightarrow[\text{law}]{}$

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Theorem:

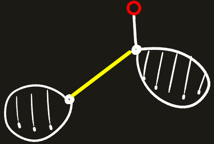
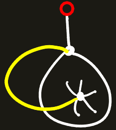
$$\boxed{\text{Degree of the root vertex divided by } n \xrightarrow[\text{law}]{} \text{uniform law on } [0,1]}$$

NUMBER OF LOOPS

$C(z,u,\ell)$ = generating function of maps where z counts the edges
u counts the degree of the root vertex
and ℓ counts the number of loops.

Equation:

$$C(z,u) = zu + u\,C(z,u)\,C(z,1) + u\left(2z^2\frac{\partial C}{\partial z} - zC\right) + (u^2\ell - u)\frac{\partial C}{\partial u}$$

map =  or  or 

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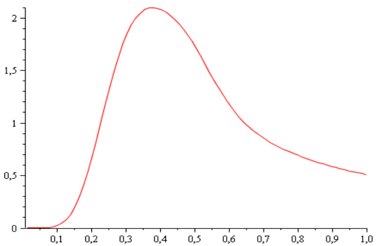
Theorem

number of loops divided by n $\xrightarrow{\text{law}}$



PART III

Lessons to learn

$\boxed{1}$ Know your bijections

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Why does the root vertex degree asymptotically follow an uniform law?

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Why does the root vertex degree asymptotically follow an uniform law?

indecomposable diagrams with n chords and k left-to-right maxima $\xleftrightarrow{\text{[Cori]}}$ maps with n edges and k vertices

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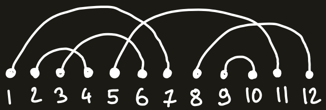

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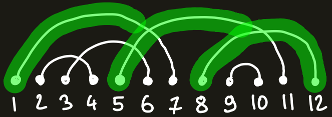

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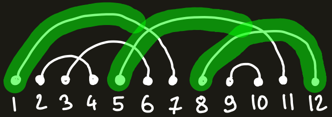

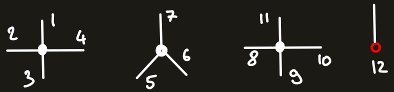

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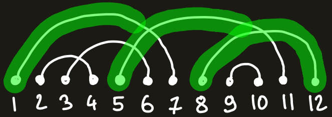

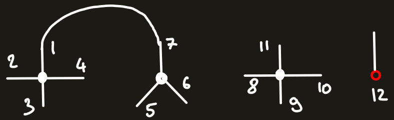

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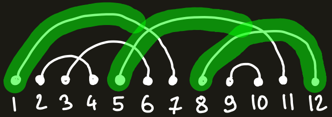

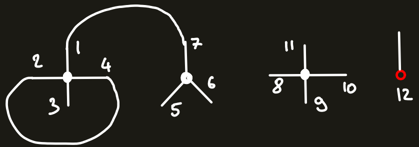

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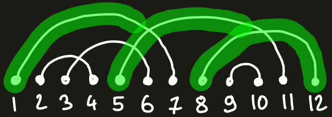

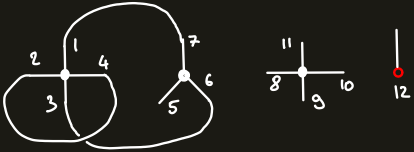

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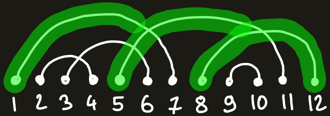

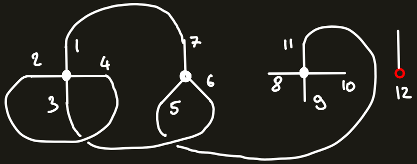

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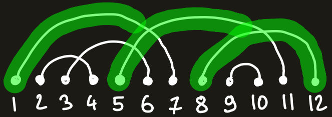

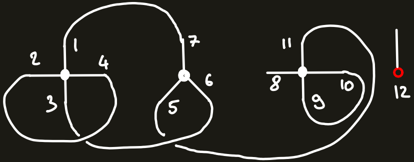

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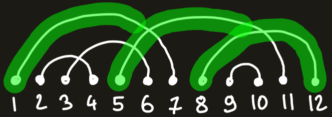

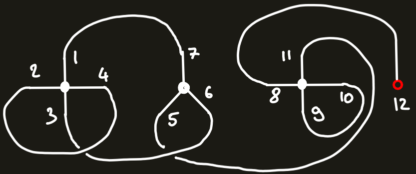

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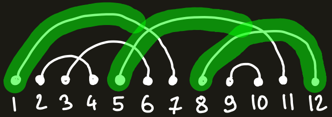

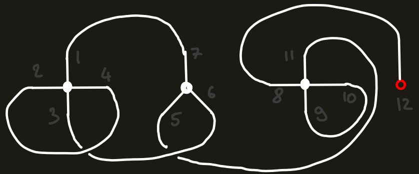

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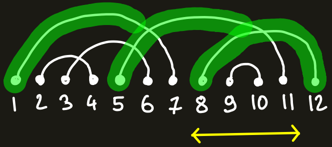
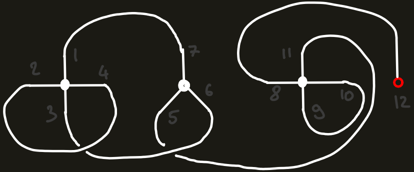


root vertex degree



$\boxed{2}$ Think bigger

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<u>Why does the magic trick work for the number of edges incident to the root?</u>

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maps $\longleftrightarrow$ indecomposable diagrams

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$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

sequence of maps $\longleftrightarrow$ diagrams

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sequence of maps $\longleftrightarrow$ diagrams

number: $(2n-1)!!$

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Why does the magic trick work for the number of edges incident to the root?

$$\begin{tikzcd}
\text{maps} \arrow[r, leftrightarrow] \arrow[d, hook] & \text{indecomposable diagrams} \arrow[d, hook] \\
\text{sequence of maps} \arrow[r, leftrightarrow] & \text{diagrams}
\end{tikzcd}$$

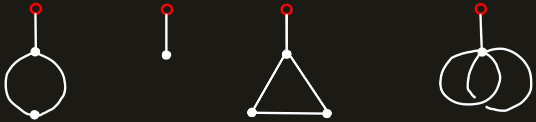
number: $(2n-1)!!$

If m_n = number of sequences $\mathrm{Map}_1, \mathrm{Map}_2, \dots, \mathrm{Map}_\ell$ with n edges,

$m_n = (2n-1) \times m_{n-1}$: explanation?

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why ?



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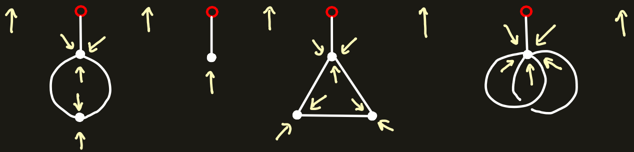
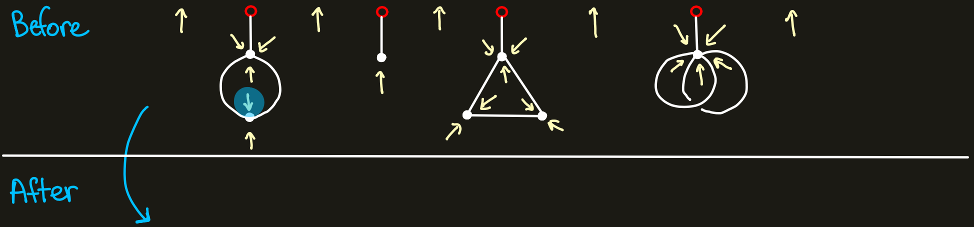


$(2n-1)$ is the number of $\uparrow$

$\boxed{2}$ Think bigger

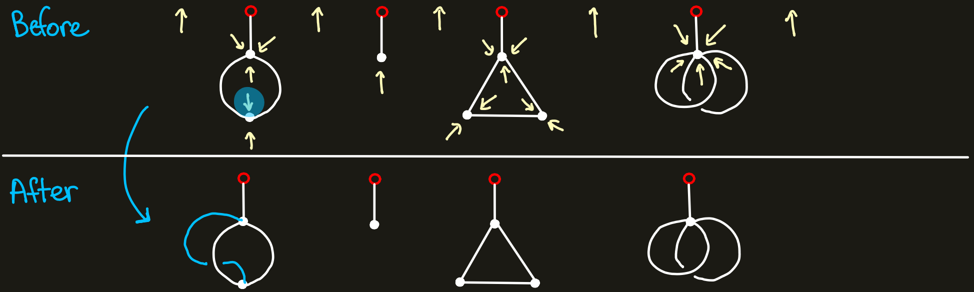
$m_n = (2n-1) \times m_{n-1}$: Why?



First case: We choose ↑ in the first map.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?



First case: We choose $\uparrow$ in the first map.

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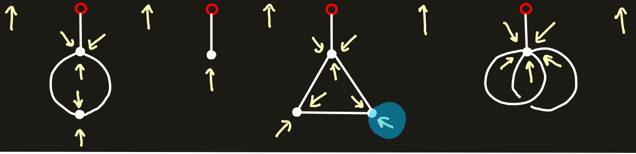
$m_n = (2n-1) \times m_{n-1}$: Why?

Before



After

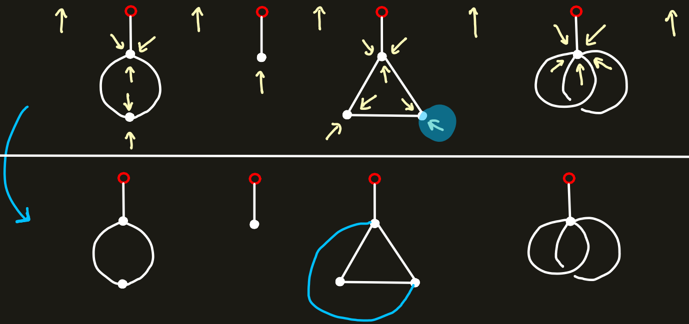
Real first case: We choose $\uparrow$ in a map.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?

Before

After



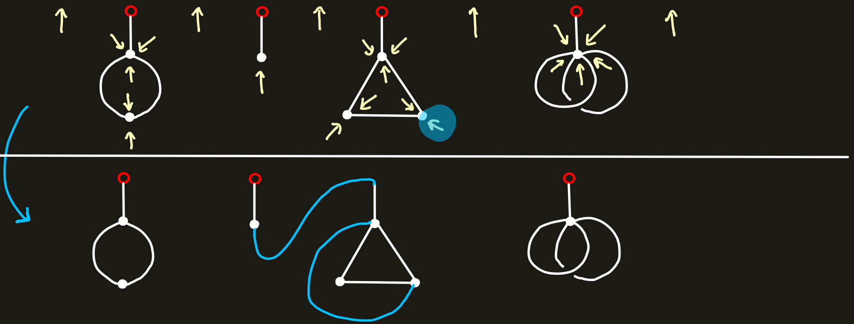
Real first case: We choose ↑ in a map.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?

Before

After



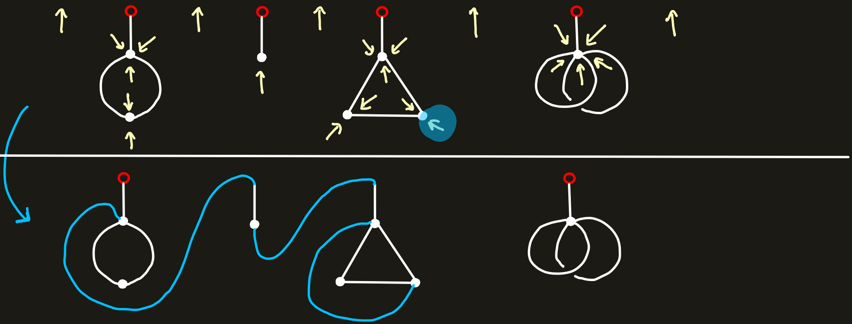
Real first case: We choose ↑ in a map.
∃

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?

Before

After

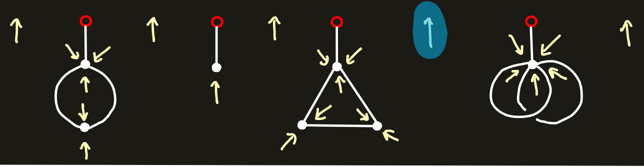


Real first case: We choose $\uparrow$ in $\underset{\exists}{a}$ map.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?

Before

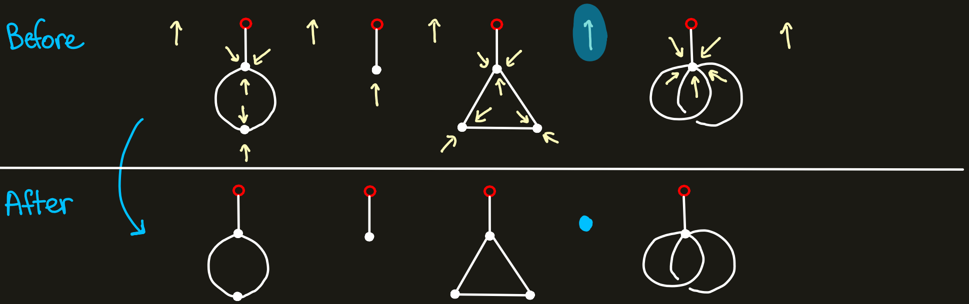


After

Second case: We choose $\uparrow$ in an interval but not the 1st.

$\boxed{2}$ Think bigger

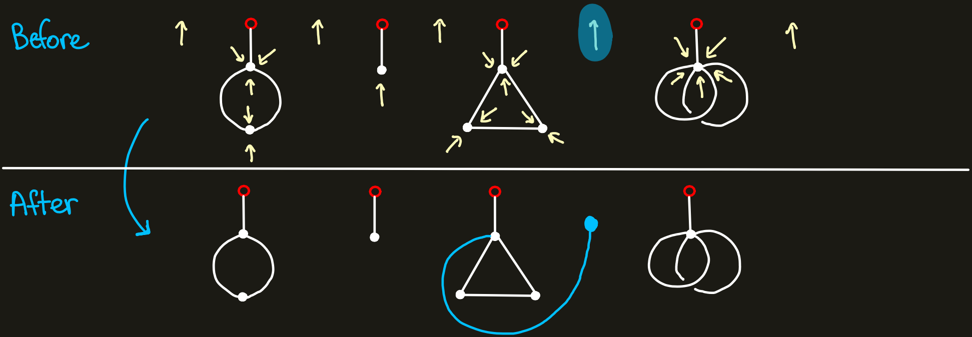
$m_n = (2n-1) \times m_{n-1}$: Why?



Second case: We choose ↑ in an interval but not the 1st.

$\boxed{2}$ Think bigger

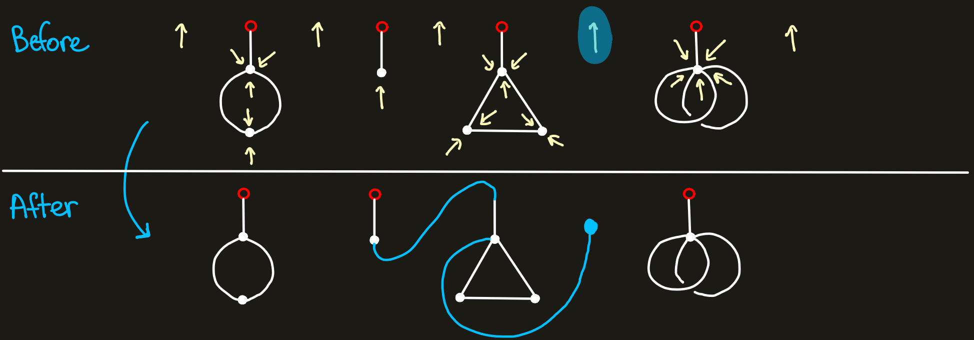
$m_n = (2n-1) \times m_{n-1}$: Why?



Second case: We choose $\uparrow$ in an interval but not the 1st.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why ?

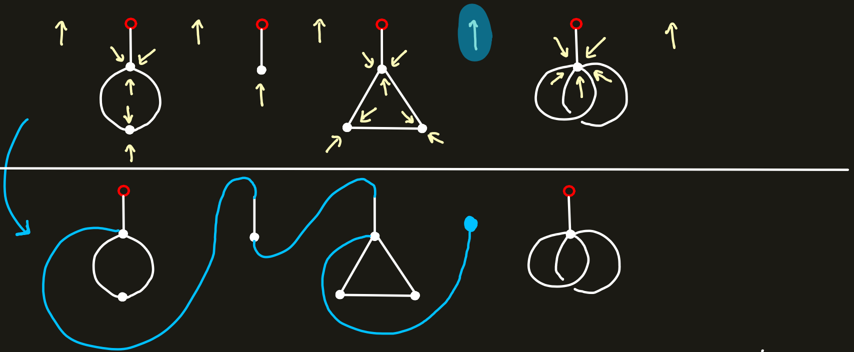


Second case: We choose ↑ in an interval but not the 1st.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?

Before

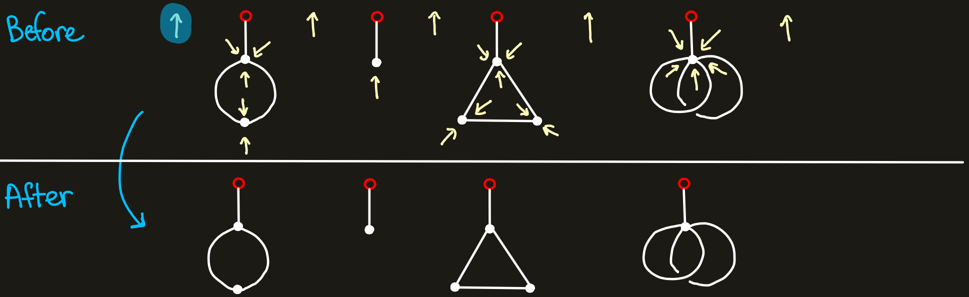


After

Second case: We choose $\uparrow$ in an interval but not the 1[st].

$\boxed{2}$ Think bigger

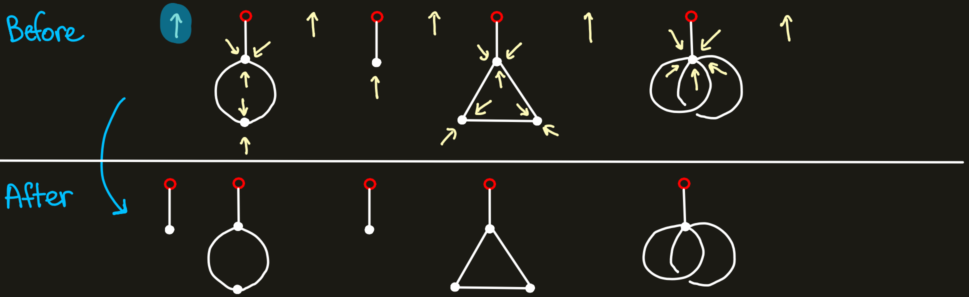
$m_n = (2n-1) \times m_{n-1}$: Why?



Last case: We choose ↑ in the first interval.

$\boxed{2}$ Think bigger

$m_n = (2n-1) \times m_{n-1}$: Why?



Last case: We choose ↑ in the first interval.

$\boxed{2}$ Think bigger

$m_n = (2n - 1) \times m_{n-1}$: Why?

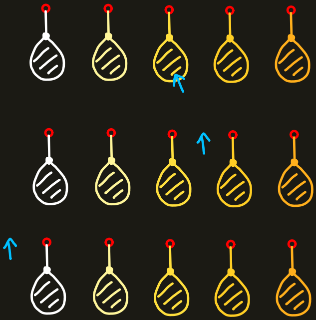
Summary:

Before

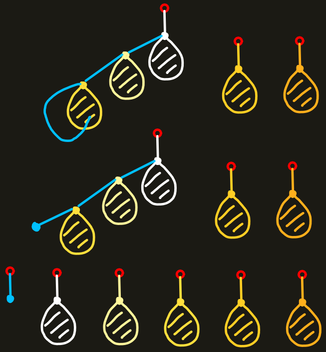
1st case:

2nd case:

3rd case:



After



$\boxed{2}$ Think bigger

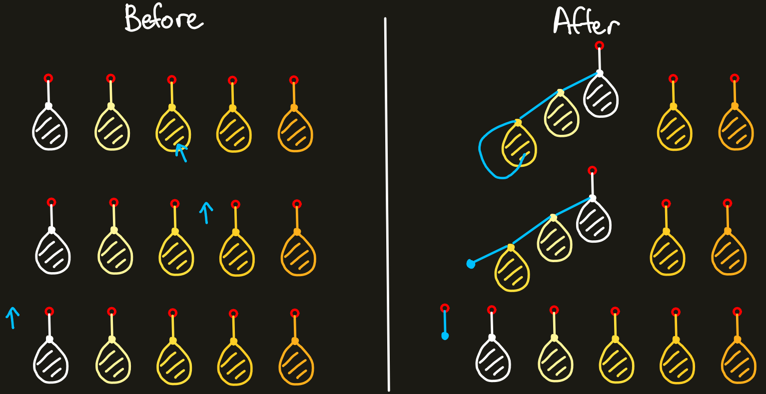
Every case except the 3rd one increases the number of edges incident to the root by 1.

Summary:

1st case:

2nd case:

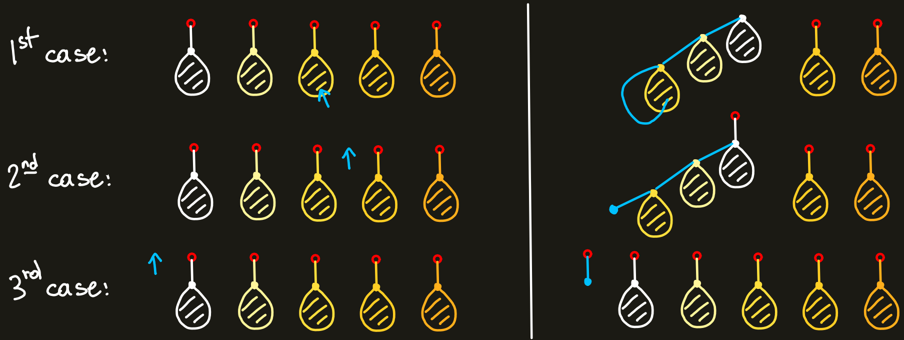
3rd case:



2 Think bigger

Every case except the 3rd one increases the number of edges incident to the root by 1.

In terms of GFs, it translates $(1-2uz)\,P(z,u) = 2uz^2\,P'(z,u) + \phi(z,1)$.



$\boxed{3}$ Be humble and work, grasshopper.

$\boxed{3}$ Be humble and work, grasshopper.

→ Wide range of limits laws for combinatorial maps:
towards a taxonomy of possible laws?

→ Understand the operation $C = z + K z^2 \frac{\phi'}{\phi}$

→ Extension to other families of maps?
to other combinatorial families?

THANK YOU!