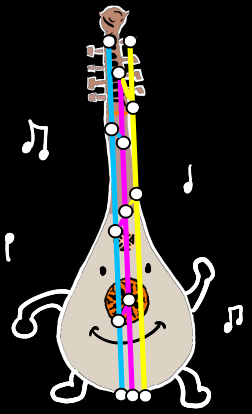
Random generation of Git graphs

Julien COURTIEL (Université de Caen Normandie)
with Martin PEPIN (Université de Caen Normandie)

Instrument beginning by "git" but not a guitar (answer at the end)



GASCom 2024

QUESTION: What do you use to share files with your coauthors?

THE TIER LIST

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good

trash

spawn of the devil

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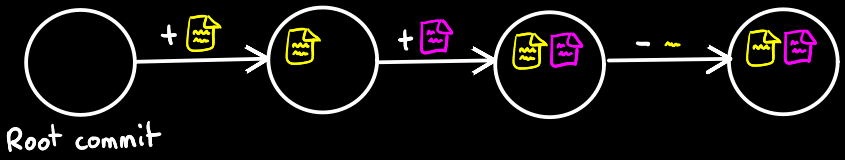
GIT FEATURES

git is a Version Control System (VCS): it stores all the project states over time.



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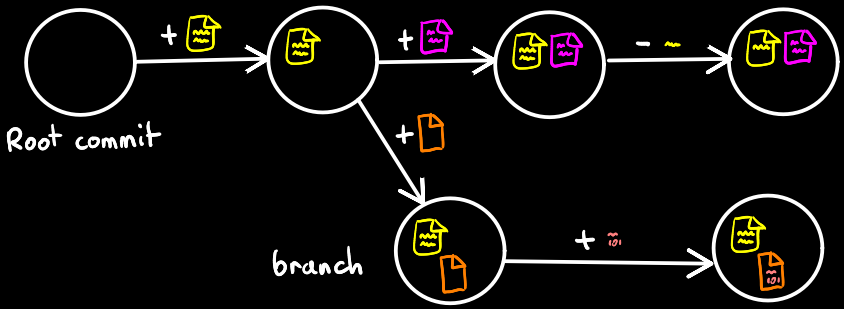

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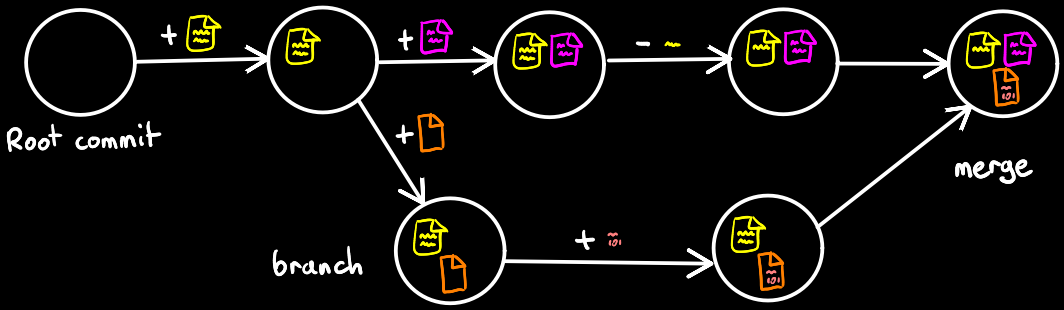

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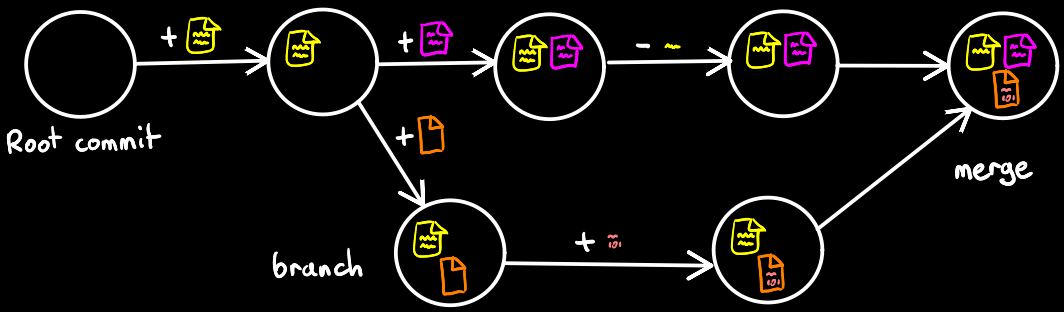

Git lets you create parallel development branches
that can be integrated later.

This forms a Directed Acyclic Graph (DAG),
where the vertices are the project states, also named commits.

OUR MOTIVATION

Objective : Design random generators for graphs of commits

Why?

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1. VCS unit testing

2. Benchmarks for VCS algorithms

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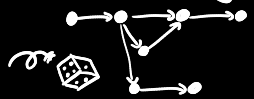
use random graphs



to check program correctness

```
void test_remove_last_commit() {

 srand(time(NULL));

 for (int i = 0; i < NUM_TESTS; i++) {

  GitGraph graph1 = create_random_git_graph();
  size_t n1 = graph1.num_commits;

  GitGraph graph2 = remove_last_commit(graph1);
  size_t n2 = graph2.num_commits;

  assert(n1 == n2 + 1);

 }
}
```

2. Benchmarks for VCS algorithms

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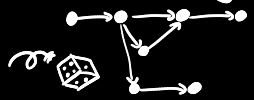

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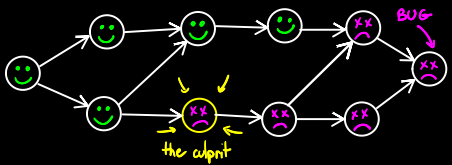
```

2. Benchmarks for VCS algorithms

e.g.

Regression Search Problem

Find the commit that introduces a bug

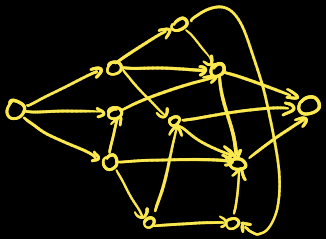


Analysis of algorithms such as git bisect

[C. Dorbec Lecoq 2023]

WHICH GRAPHS TO GENERATE?

In git, every DAG without restriction can be generated...



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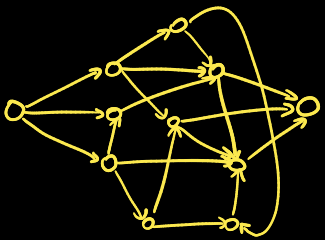

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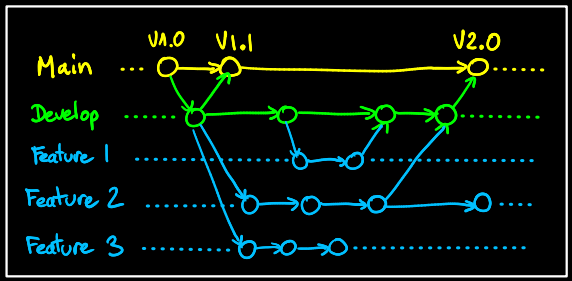

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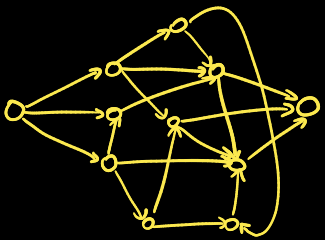

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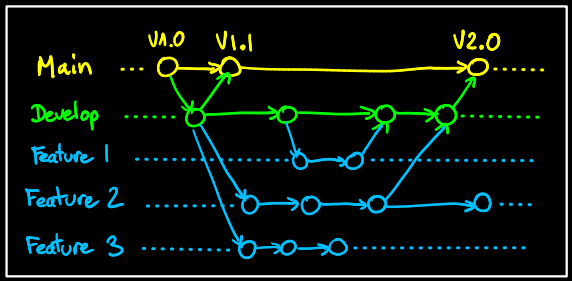


In this work, we consider a simple workflow but widely used in industry: the feature branch workflow

GIT GRAPH

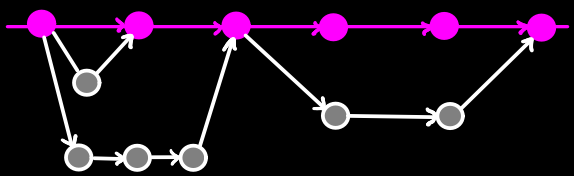
DEFINITION

(feature branch) Git graph = DAG with

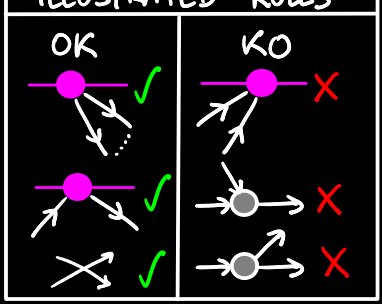
- a main branch (path of magenta vertices)
- 0, 1 or several feature branches, paths of ≥ 1 white vertices starting and ending on magenta vertices
- indegree ≤ 2 for all vertices

previously defined in [Lecoq 2024]

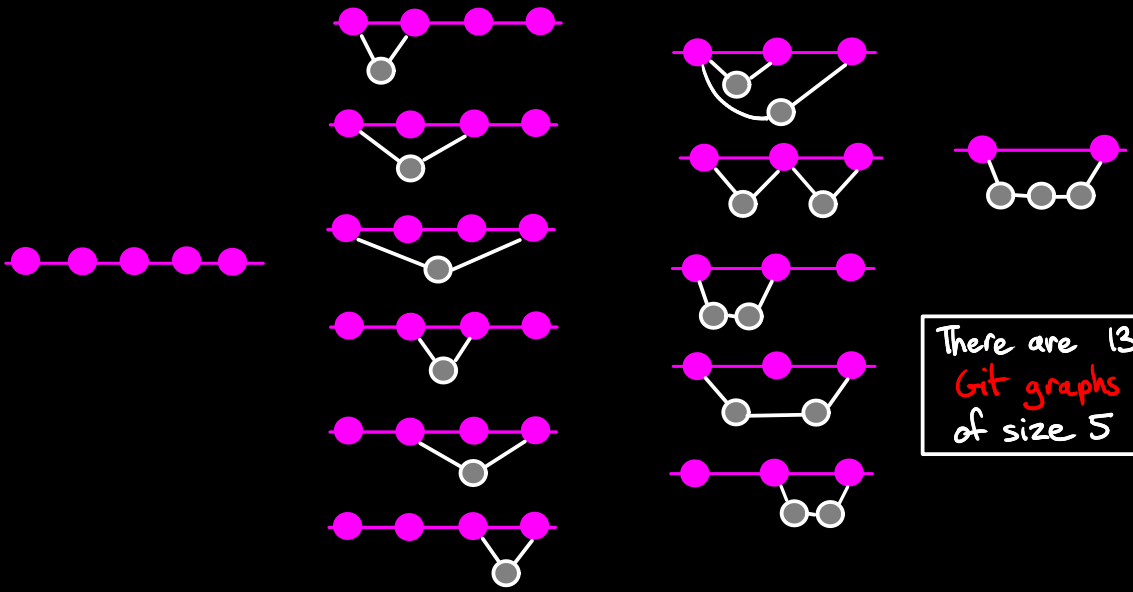
e.g.



ILLUSTRATED RULES



ENUMERATING & SAMPLING



There are 13 Git graphs of size 5

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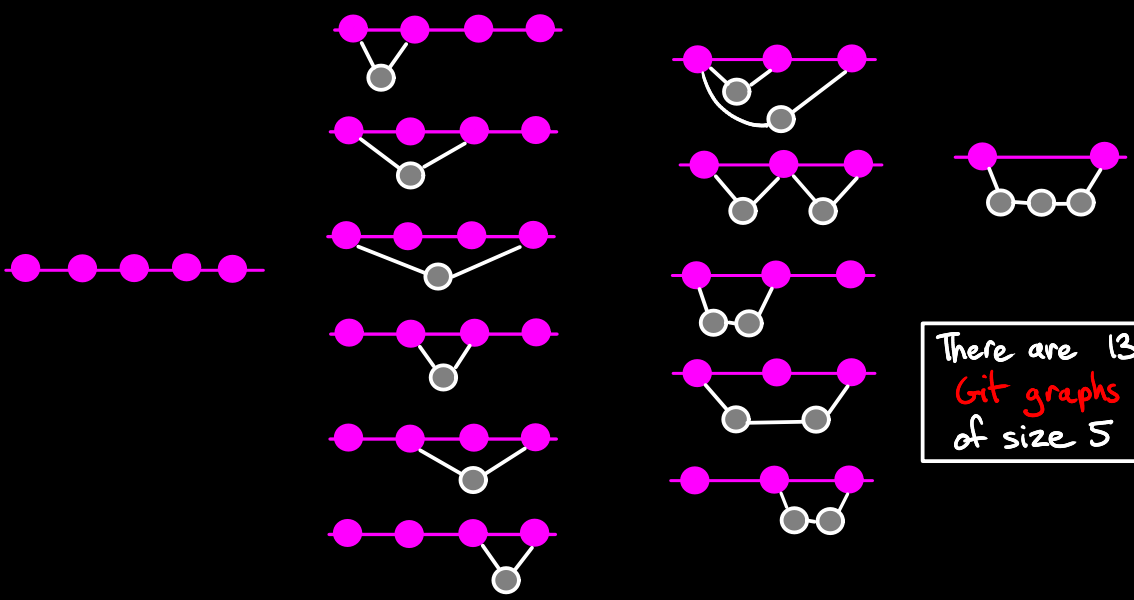

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GOAL

Sample a Git graph uniformly at random
given a size n and a number k of magenta vertices

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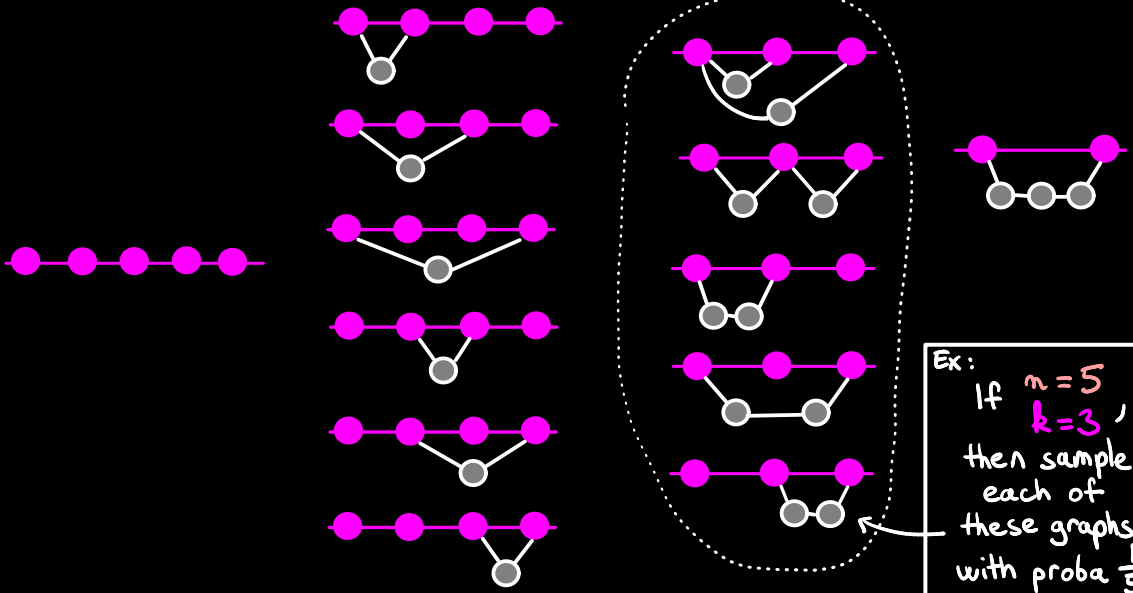

Ex: If $n=5$, $k=3$, then sample each of these graphs with proba $\frac{1}{5}$

GOAL

Sample a Git graph uniformly at random
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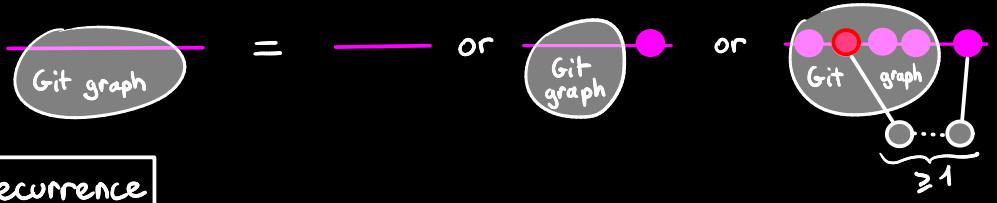
Decomposition

RECURSIVE DECOMPOSITION



Recurrence

$$g_{n,k} = g_{n-1,k-1} + \sum_{l \geq 1} (k-1)\, g_{n-1-l,k-1} \qquad \text{for } n \geq 1$$

where $g_{n,k}$:= number of Git graphs with n vertices, k of them being magenta

Differential Equation for the Generating Function

$$G(z,u) = 1 + zu\,G(z,u) + \frac{z^2u^2}{1-z}\frac{\partial G}{\partial u}(z,u)$$

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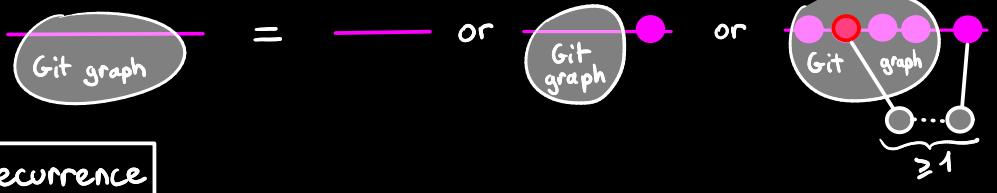

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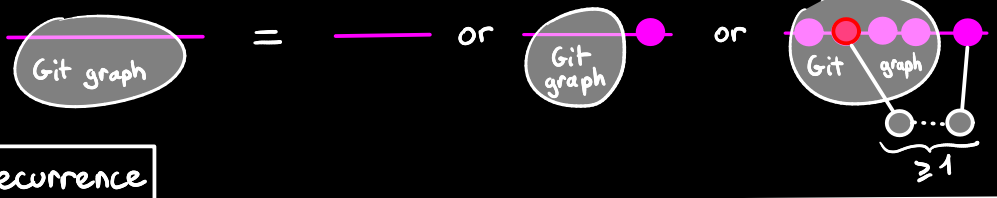

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→ Bolzmann sampling impossible (?) since $G(z,u)$ not analytic

TRANSFORMING THE EQUATION

Recurrence	$g_{m,k} = g_{m-1,k-1} + \sum_{l \geq 1} (k-1)\, g_{m-1-l,k-1}$
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Usual trick:

Ordinary Generating Function $\underbrace{\sum_{m,k \geq 0} g_{m,k} z^m u^k}_{G(z,u),\ \text{not analytic}\ \times}$

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Borel transform →

Exponential Generating Function: $\underbrace{\sum_{n,k \geq 0} \frac{g_{n,k}}{n!} z^n u^k}_{\text{analytic}}$

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Exponential Generating Function $\sum_{n,k \geq 0} \frac{g_{n,k}}{n!} z^n u^k$ analytic, but no pretty equation ✗

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$$\tilde{G}(z,u) = \sum_{n,k \geq 0} \frac{g_{n,k}}{k!} z^n u^k$$

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Borel transform on u

$$\tilde{G}(z,u) = \sum_{n,k\geq 0} \frac{g_{n,k}}{k!} z^n u^k$$

analytic ✓

and

Differential Equation for $\tilde{G}$
$\frac{\partial \tilde{G}}{\partial u} = z\tilde{G} + \frac{z^2 u}{1-z}\frac{\partial \tilde{G}}{\partial u}$ ✓

TRANSFORMING THE EQUATION

$\tilde{G}(z, u)$ = Exponential (in u) Generating Function of Git graphs

z counts vertices u counts magenta vertices

$$\tilde{G}(z,u) = \sum_{m,k \geq 0} g_{m,k} \frac{z^m u^k}{k!}$$

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this can be solved!

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$\tilde{G}(z_g, u)$ = Exponential (in u) Generating Function of Git graphs

z_g counts vertices u counts magenta vertices

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analytic ✓

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Differential Equation for $\tilde{G}$

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this can be solved!

Theorem

$$\tilde{G}(z_g, u) = \left(1 - \frac{z_g^2 u}{1 - z_g}\right)^{-\frac{1 - z_g}{z_g}}$$

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How can it be exploited?

RANDOM GENERATION?

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"Boltzmann model" (exponential in u, ordinary in z)

Fix $z > 0^*$ and $u > 0^*$.

We wish to sample a Git graph $\mathcal{G}$ with a weight proportional to $z^{\#\text{vertices in } \mathcal{G}} \dfrac{u^{\#\text{magenta vertices in } \mathcal{G}}}{(\#\text{magenta vertices in } \mathcal{G})!}$

(Size is not fixed)

Examples

$\mathbb{P}(\text{—}) \propto 1$ | $\mathbb{P}(\text{—●—}) \propto z\,u$ | $\mathbb{P}(\text{●●●●}) \propto z^5 \frac{u^4}{24}$ | $\mathbb{P}(\text{●●●●}) \propto z^5 \frac{u^3}{6}$

*: in the disk of convergence of $\tilde{G}$

RANDOM GENERATION?

"Boltzmann model" (exponential in u, ordinary in z_γ)

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where $\widetilde{G}(z_\gamma, u) = \sum_{n,k \geq 0} g_{n,k} \dfrac{z_\gamma^n u^k}{k!} = \left(1 - \dfrac{z_\gamma^2 u}{1 - z_\gamma}\right)^{-\frac{1-z_\gamma}{z_\gamma}}$.

Examples

$\mathbb{P}(\text{—}) = \dfrac{1}{\widetilde{G}(z_\gamma, u)}$ $\quad$ $\mathbb{P}(\text{—●—}) = \dfrac{z_\gamma u}{\widetilde{G}(z_\gamma, u)}$ $\quad$ $\mathbb{P}(\text{●—●—●}) = \dfrac{z_\gamma^5}{\widetilde{G}(z_\gamma, u)} \dfrac{u^4}{24}$ $\quad$ $\mathbb{P}(\text{●—●—●}) = \dfrac{z_\gamma^5}{\widetilde{G}(z_\gamma, u)} \dfrac{u^3}{6}$

*: in the disk of convergence of $\widetilde{G}$

CHOOSING z AND u

Proposition

Let γ be a random Git Graph sampled with respect to the previous Boltzmann model, conditioned to have size n

$$\mathbb{E}(\#\text{magenta vertices}(\gamma)) \sim \frac{1-\rho_u}{2-\rho_u} n$$

$$\mathbb{V}(\#\text{magenta vertices}(\gamma)) \sim \frac{\rho_u(1-\rho_u)}{(2-\rho_u)^3} n$$

where $\rho_u = \dfrac{\sqrt{1+4u}-1}{2u}$

Proof : Transfer Theorem from $\tilde{G}(z,u) = \left(1 - \frac{z^2 u}{1-z}\right)^{-\frac{1-z}{z}}$

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Consequence : Given any $\alpha \in (0, \frac{1}{2})$, and $n \geq 0$, we can tune u to target αn magenta vertices and then z_γ to target size n.

Different Perspective

Is there a combinatorial explanation for the formula

$$\widetilde{G}(z_\gamma, u) = \left(1 - \frac{z_\gamma^2 u}{1 - z_\gamma}\right)^{-\frac{1 - z_\gamma}{z_\gamma}} ?$$

Different Perspective

Is there a combinatorial explanation for the formula

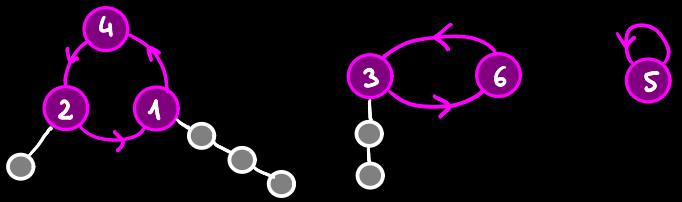
$$\widetilde{G}(z_\delta, u) = \left(1 - \frac{z_\delta^2 u}{1 - z_\delta}\right)^{-\frac{1-z_\delta}{z_\delta}} = \exp\left(\frac{1}{\frac{z_\delta}{1-z_\delta}} \ln\left(\frac{1}{1 - u z_\delta \frac{z_\delta}{1-z_\delta}}\right)\right)?$$

DIFFERENT PERSPECTIVE

Definition

cyclarium = set of cycles of magenta vertices labeled from 1 to k where a chain of white unlabeled vertices is attached to each magenta vertex, except to the ones having the largest label in their cycles.

e.g.:



Is there a combinatorial explanation for the formula

$$\widetilde{G}(z_0, u) = \left(1 - \frac{z_0^2 u}{1 - z_0}\right)^{-\frac{1-z_0}{z_0}} = \exp\left(\frac{1}{\frac{z_0}{1-z_0}} \ln\left(\frac{1}{1 - u z_0 \frac{z_0}{1-z_0}}\right)\right) ?$$

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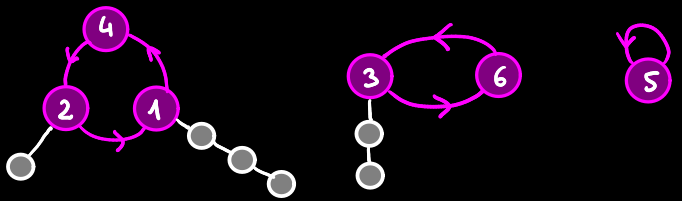
cyclarium = set of cycles of magenta vertices labeled from 1 to k where a chain of white unlabeled vertices is attached to each magenta vertex, except to the ones having the largest label in their cycles.

e.g.:



Is there a combinatorial explanation for the formula

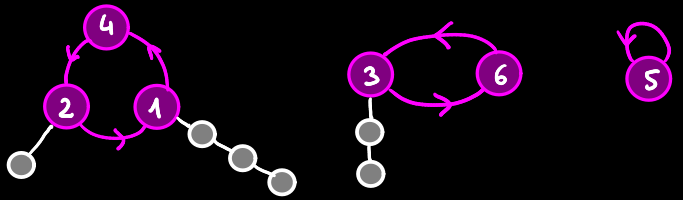
$$\widetilde{G}(z_0, u) = \left(1 - \frac{z_0^2 u}{1 - z_0}\right)^{-\frac{1-z_0}{z_0}} = \exp\left(\frac{1}{\frac{z_0}{1-z_0}} \ln\left(\frac{1}{1 - u z_0 \frac{z_0}{1-z_0}}\right)\right) ?$$

It's the generating function of cyclariums!

BIJECTION

Proposition

There is a bijection from cyclariums to Git graphs.



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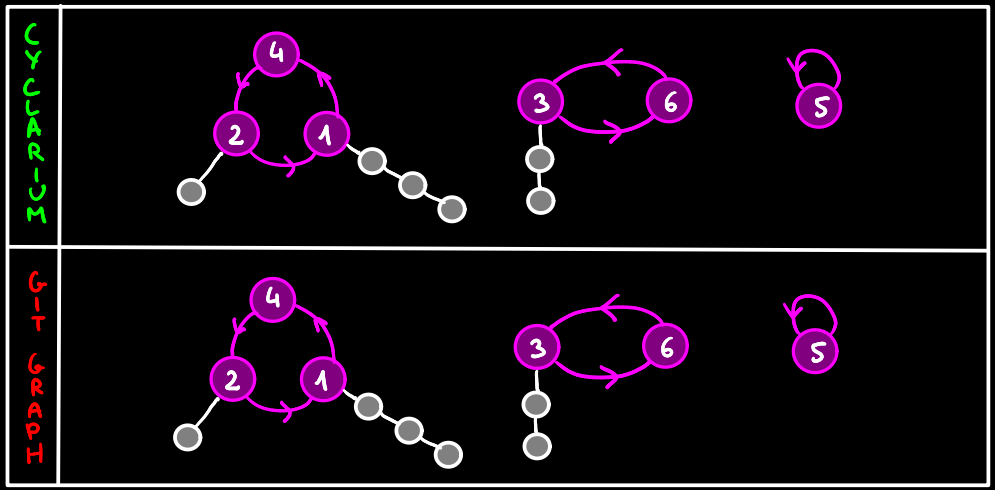

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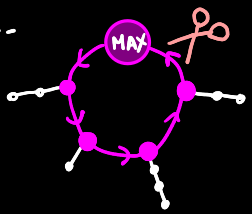

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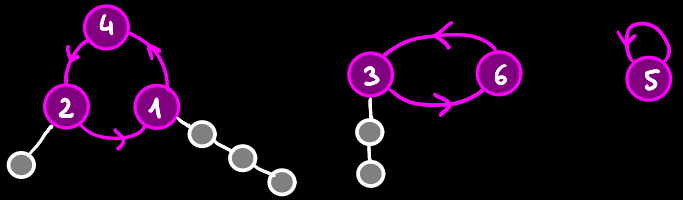

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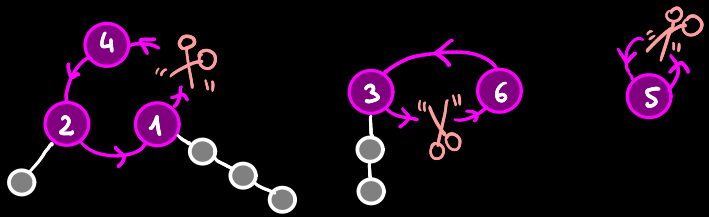

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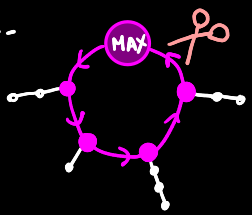

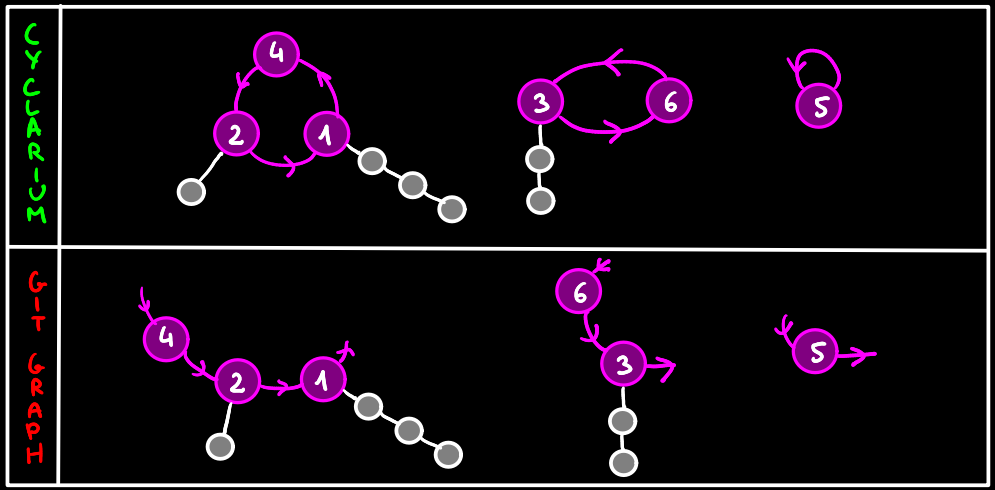

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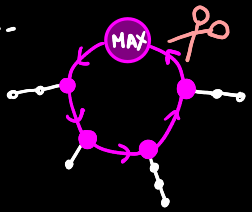

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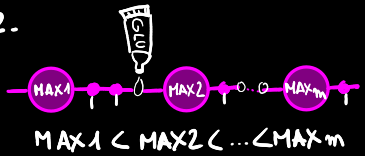

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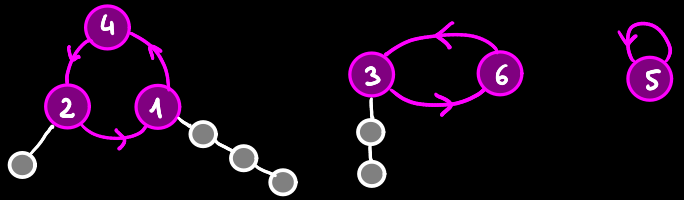

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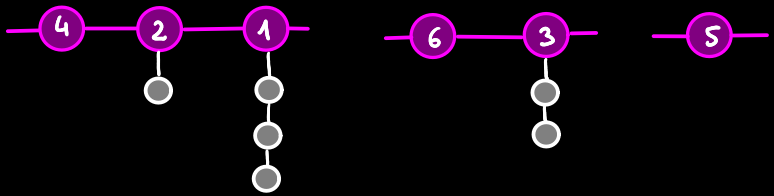

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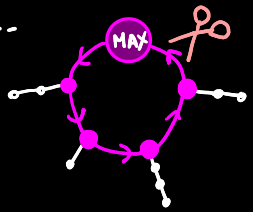

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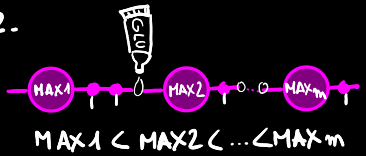

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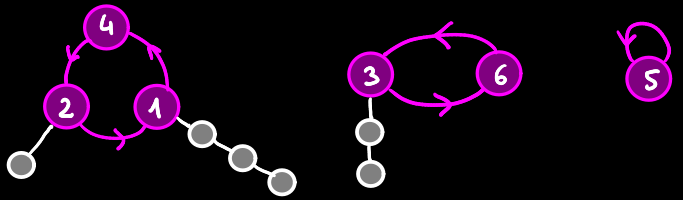

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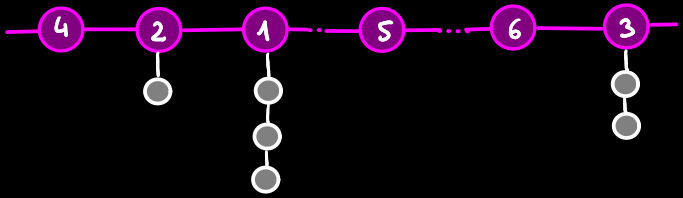

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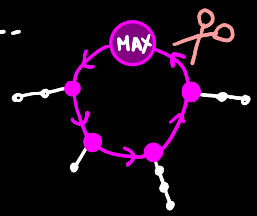

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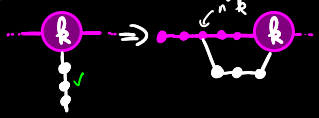
3a.



3b. Label removing + shift

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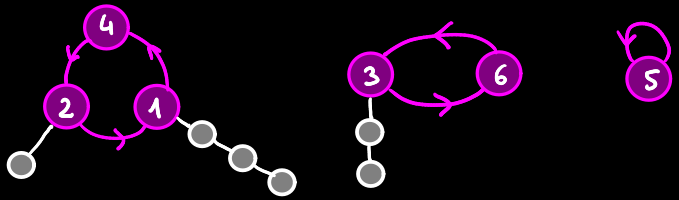

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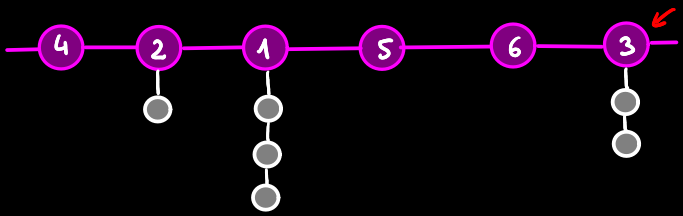

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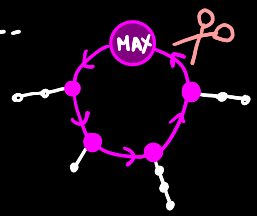

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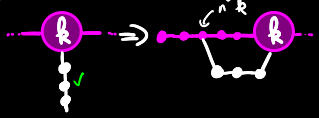

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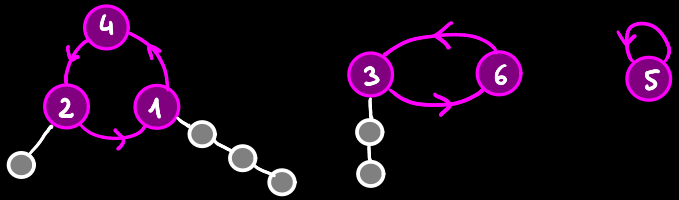

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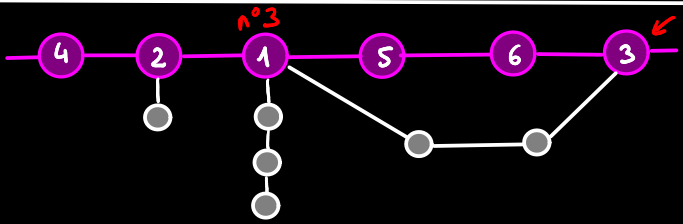

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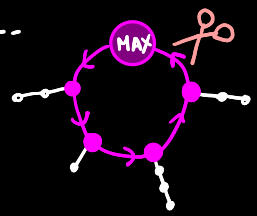

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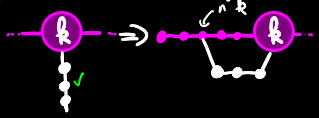

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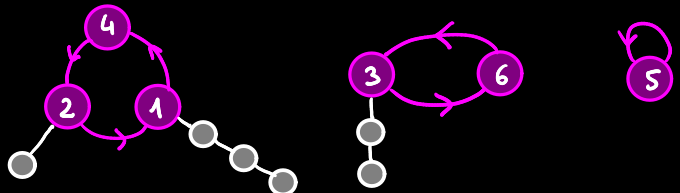

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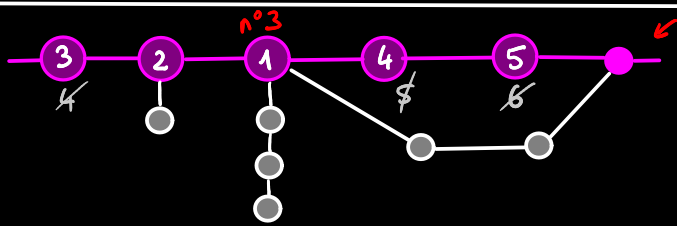

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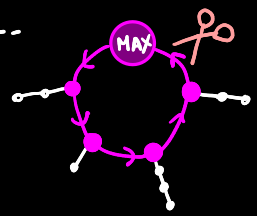

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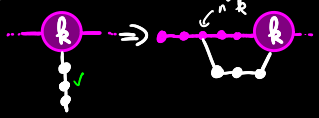

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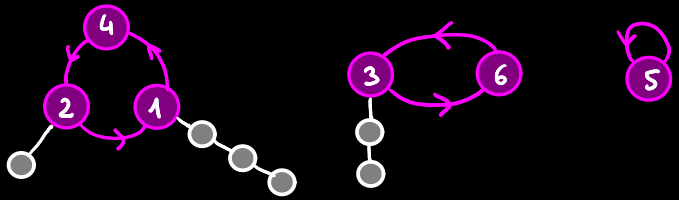

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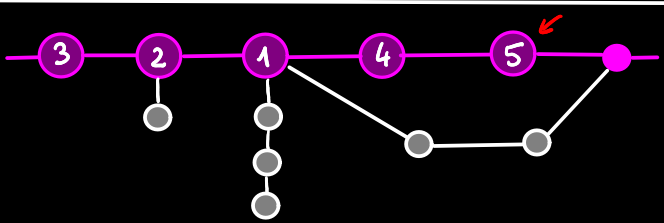

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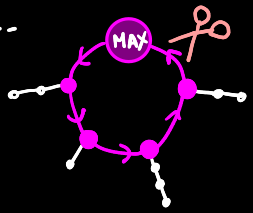
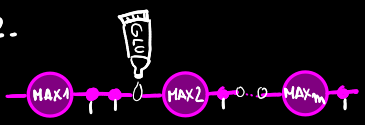
1. 

2. 

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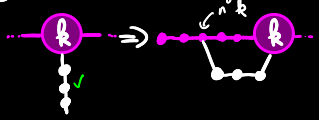
3a. 

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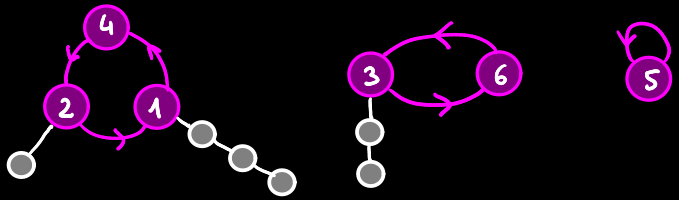

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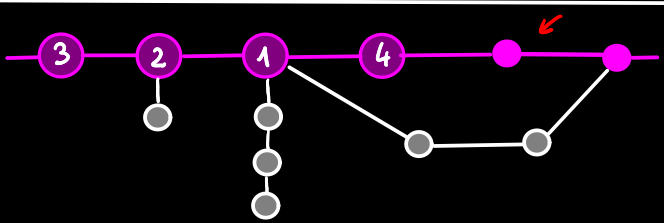

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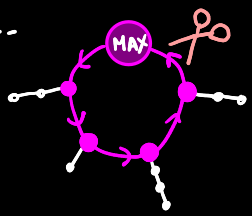
1. 

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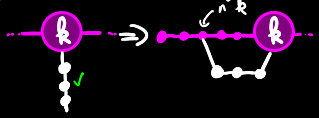
3a. 

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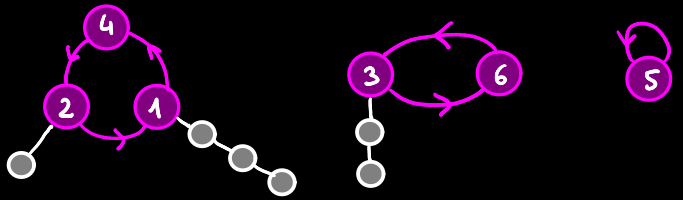

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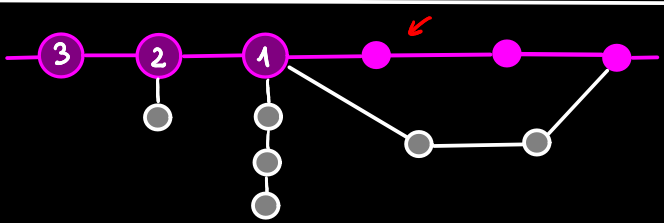

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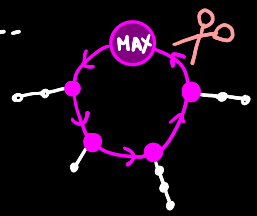

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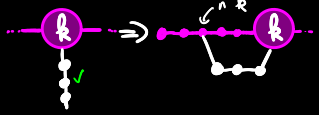

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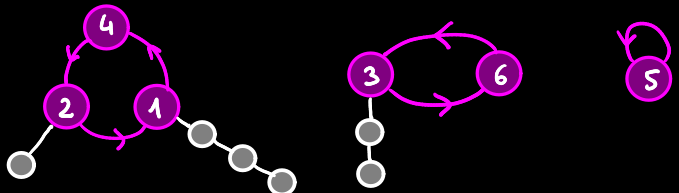

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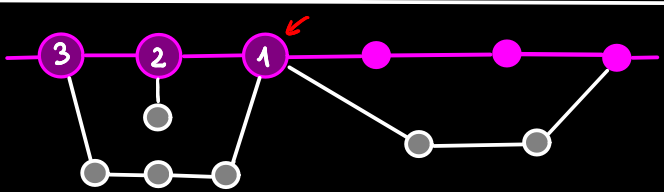

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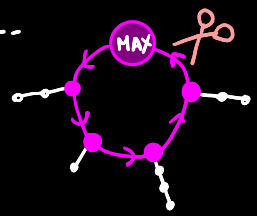

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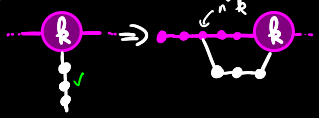

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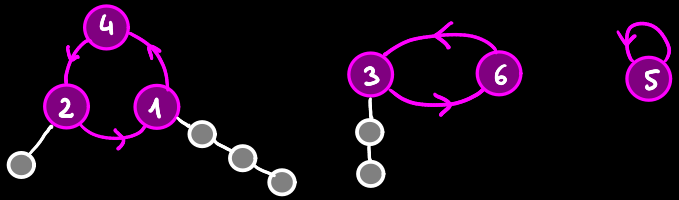

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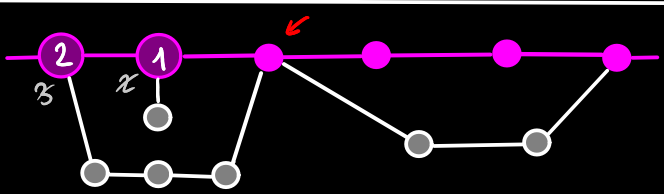

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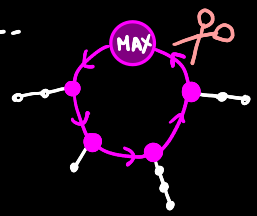

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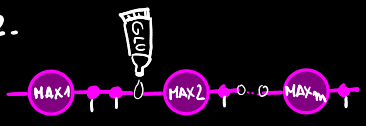

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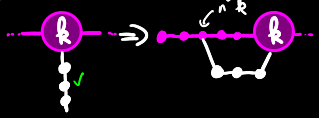

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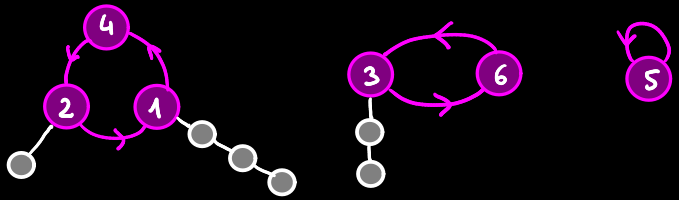

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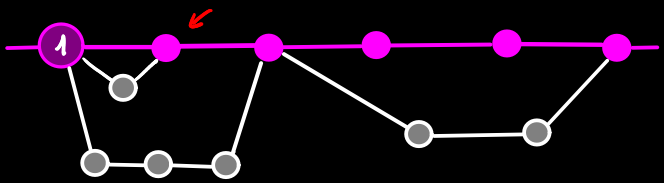

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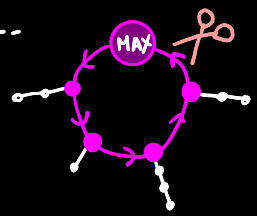
STEPS

1.

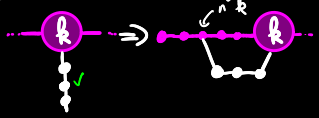


2.

MAX1 < MAX2 < ... < MAXm

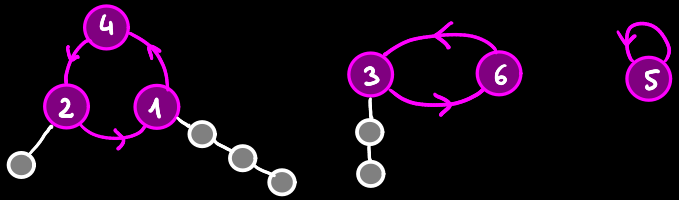
3. Right to left:

3a.

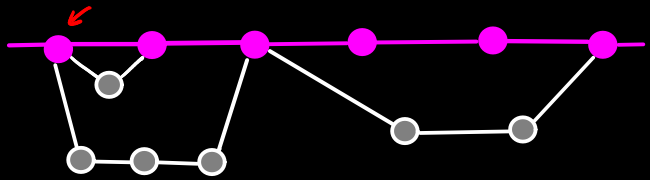


3b. Label removing + shift

CYCLARIUM



GIT GRAPH

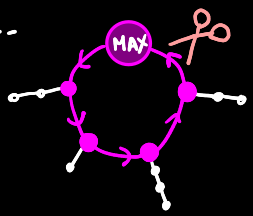


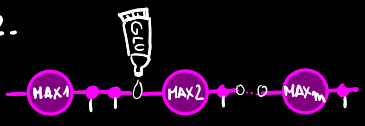
BIJECTION

Proposition

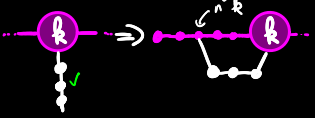
There is a bijection from cyclariums to Git graphs:

STEPS

1. 

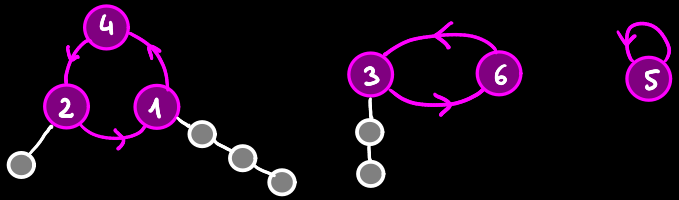
2. 

3. Right to left:

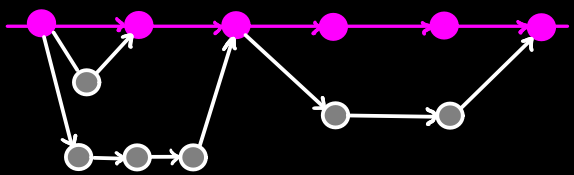
3a. 

3b. Label removing + shift

CYCLARIUM



GIT GRAPH



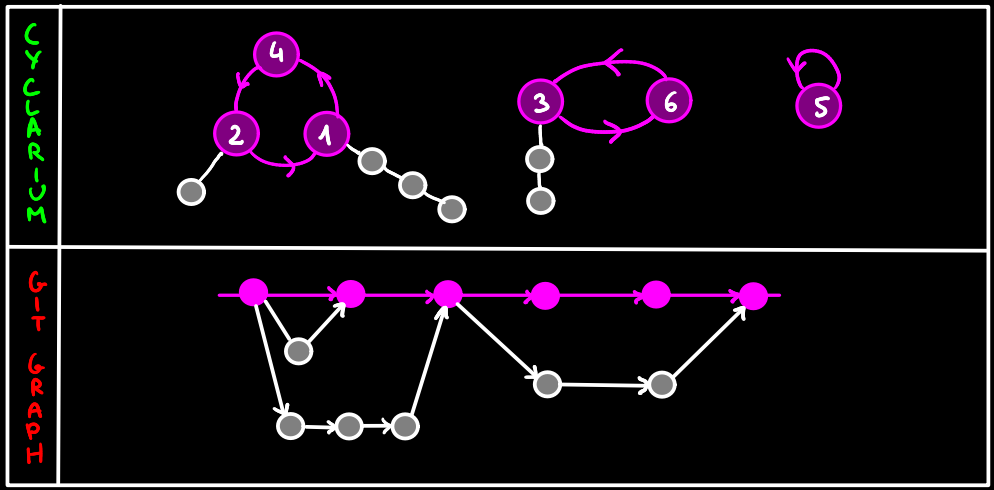
BIJECTION

Proposition

There is a bijection from cyclariums to Git graphs: sending

vertices ⟶ vertices

magenta vertices ⟶ magenta vertices

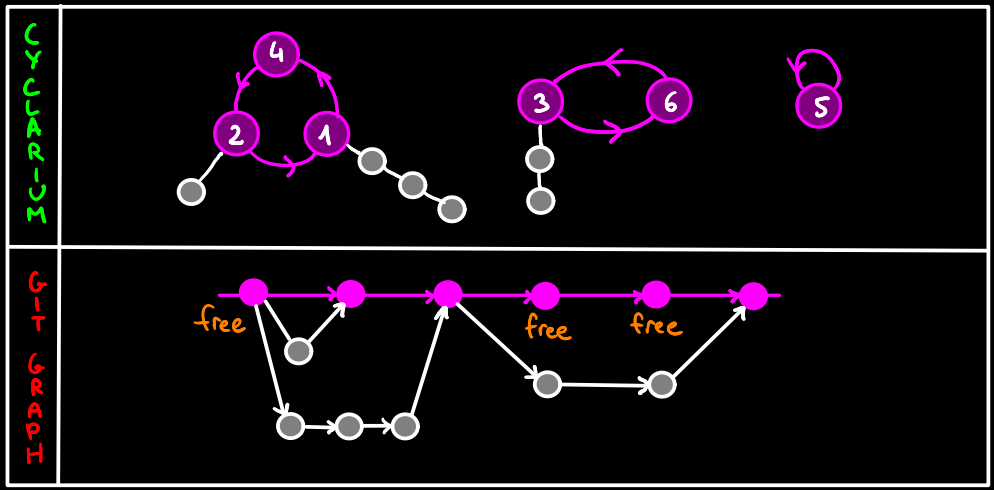


BIJECTION

Proposition

There is a bijection from cyclariums to Git graphs: sending

- vertices ⟶ vertices
- magenta vertices ⟶ magenta vertices
- cycles ⟶ free vertices / i.e magenta vertices of indegree ≤ 1
- cycle lengths ⟶ gaps between free vertices



A NICE FORMULA

Proposition

There is a bijection from cyclariums to Git graphs:
sending

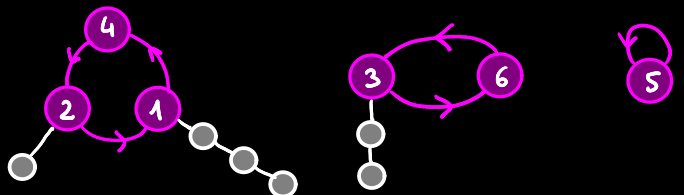
vertices ⟶ vertices

magenta vertices ⟶ magenta vertices

cycles ⟶ free vertices
i.e magenta vertices of indegree ≤ 1

cycle lengths ⟶ gaps between free vertices

CYCLARIUM



Corollary

$$g_{n,k} = \sum_{f=1}^{k-1} \begin{bmatrix} k \\ f \end{bmatrix} \binom{n-k-1}{k-f-1} \qquad (k < n)$$

where $g_{n,k}$ = number of Git graphs counted by vertices & magenta vertices
and $\begin{bmatrix} : \\ : \end{bmatrix}$ = (unsigned) Stirling number of 1st kind

BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

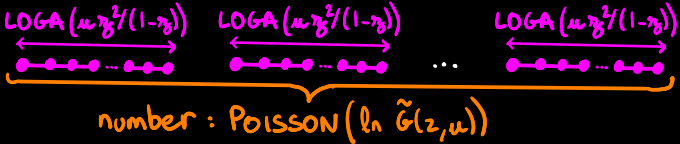
Output: A Git graph $\mathcal{G}$ with proba $z^{\text{size}(\mathcal{G})}\, u^{\#\text{magental}(\mathcal{G})} / (\#\text{magenta}(\mathcal{G})!\, \widetilde{G}(z,u))$

BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \tilde{G}(z,u))$

Step 1. Sample



Ex:

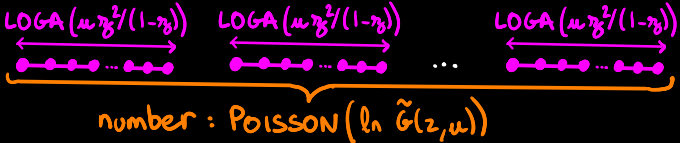
POISSON = 3

BOLTZMANN SAMPLER

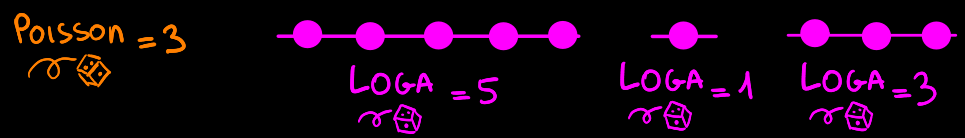
Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \tilde{G}(z,u))$

Step 1. Sample



Ex :



BOLTZMANN SAMPLER

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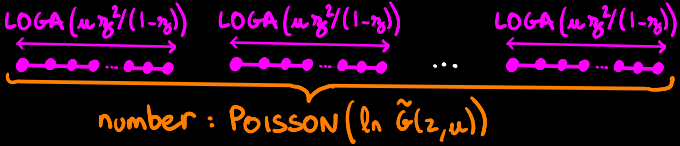
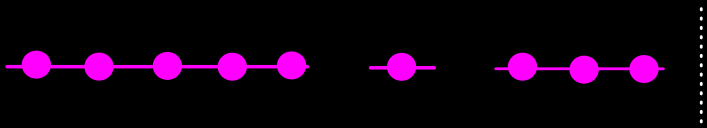
Step 1. Sample



Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Ex :



BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \tilde{G}(z,u))$

Step 1. Sample

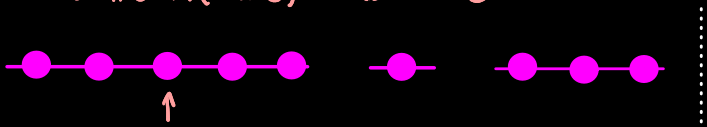
$\text{LOGA}(u z^2/(1-z))$ $\text{LOGA}(u z^2/(1-z))$ ... $\text{LOGA}(u z^2/(1-z))$

number : $\text{POISSON}(\ln \tilde{G}(z,u))$

Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Ex :

$\text{UNIFORM}(1 \dots 9)$ = 3

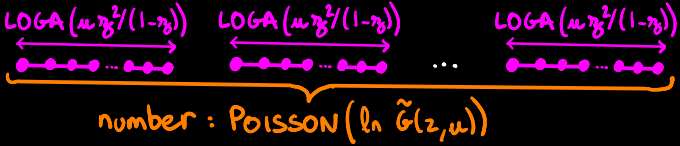


BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \tilde{G}(z,u))$

Step 1. Sample



Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

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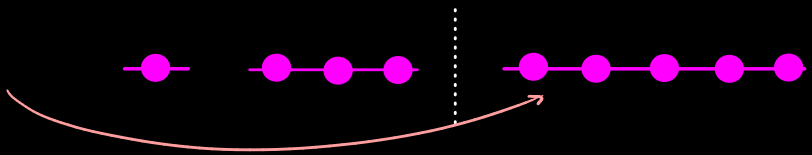

BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

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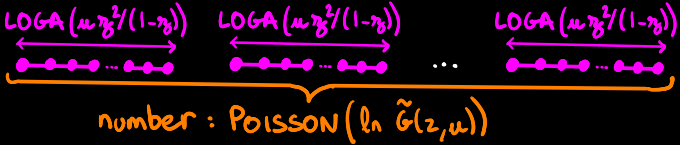

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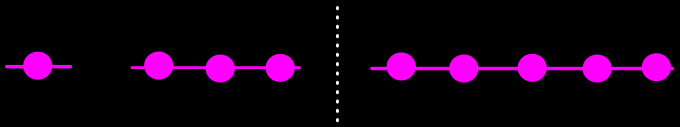

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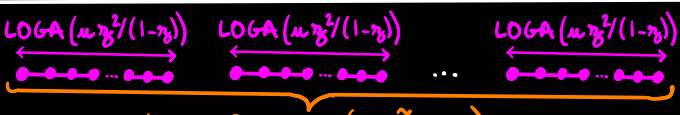
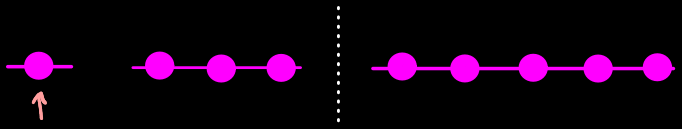


Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Ex :

UNIFORM(1...4) = 1

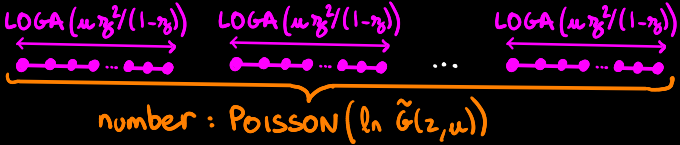


BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z,u))$

Step 1. Sample



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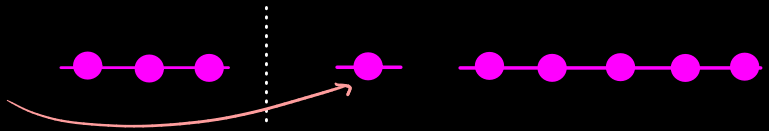

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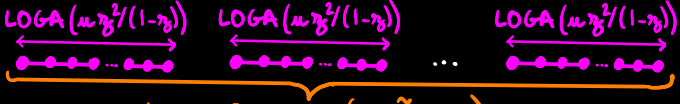

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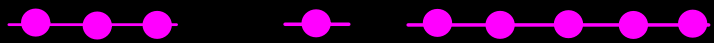

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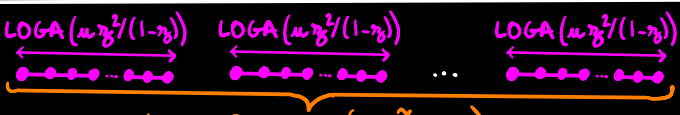

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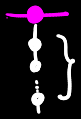
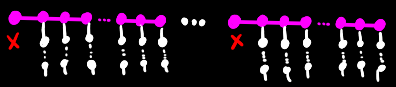
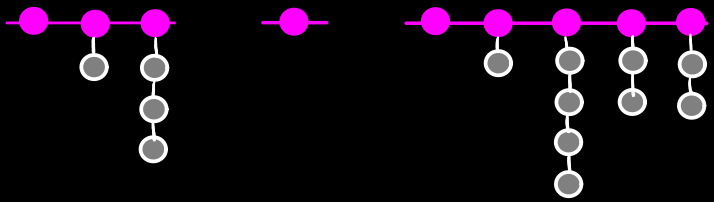
Step 3 -  $1 + \text{GEOM}(z)$ for 

Ex :

GEOM  x6

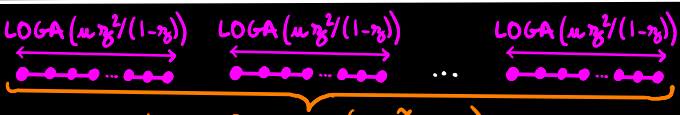


BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \tilde{G}(z,u))$

Step 1. Sample



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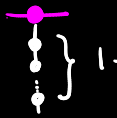
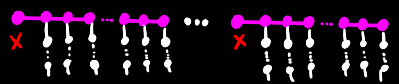
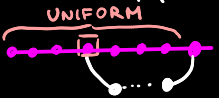
Step 3.  $1 + \text{GEOM}(z)$ for 

Step 4. Glue + for each  , 

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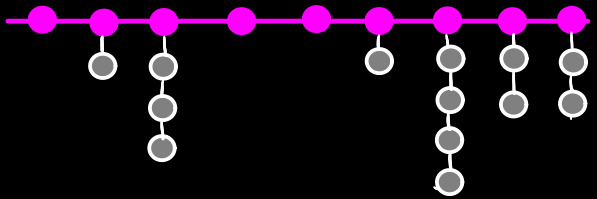

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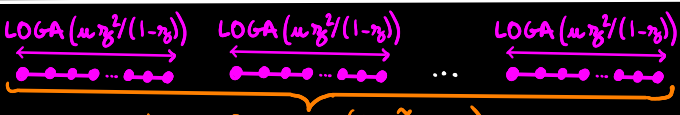

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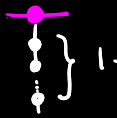
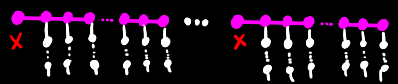
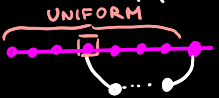
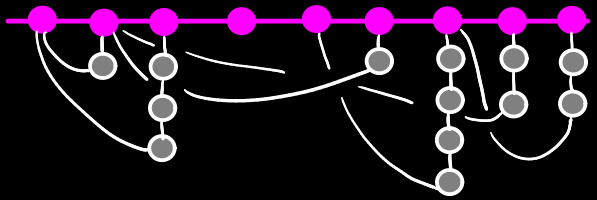
Step 3.  $1 + \text{GEOM}(z)$ for 

Step 4. Glue t for each  , 

Ex:

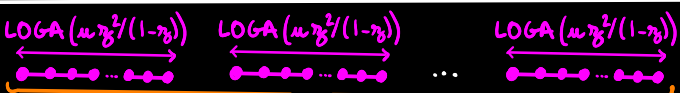


BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph δ with proba $z^{\text{size}(\delta)}\, u^{\#\text{magenta}(\delta)} / (\#\text{magenta}(\delta)!\, \tilde{G}(z,u))$

Step 1. Sample



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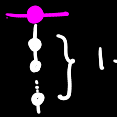
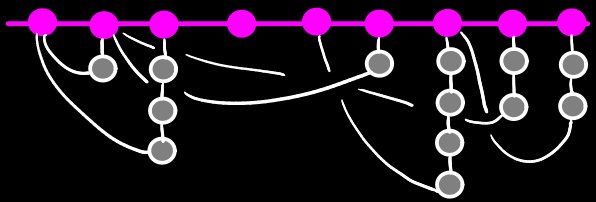
Step 3.  $1 + \text{GEOM}(z)$ for 

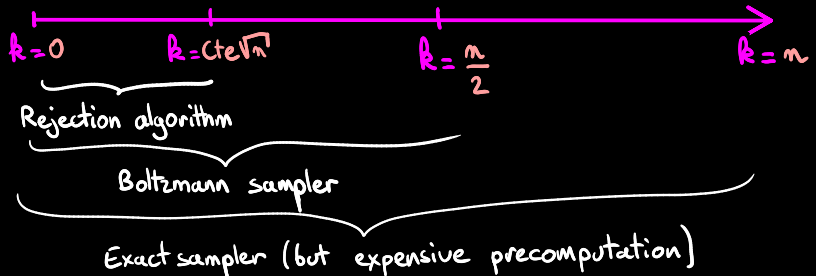
Step 4. Glue + for each  , 

Ex: 

COMPLEXITY: $O(n)$ (in average)

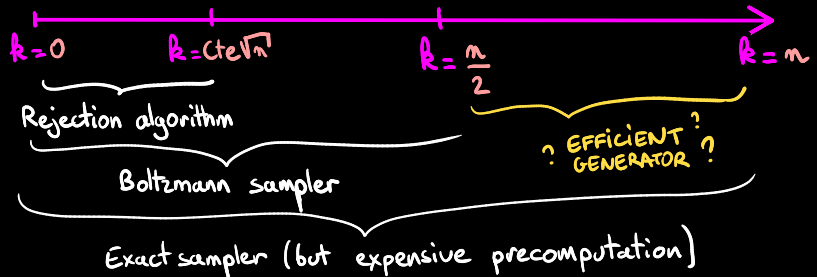
CONCLUSION

Sampling with respect to ratio $\frac{\#\text{ magenta vertices}}{\text{size}}$



CONCLUSION

Sampling with respect to ratio # magenta vertices / size

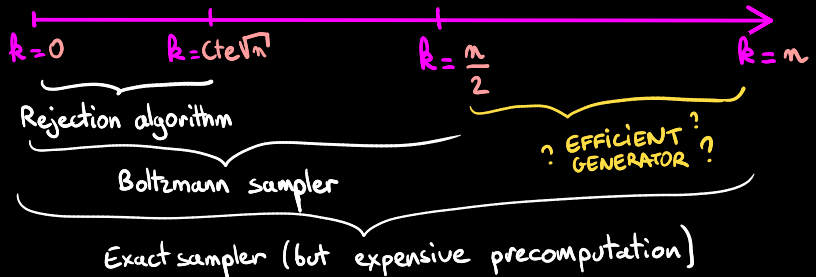


PERSPECTIVES →

→

CONCLUSION

Sampling with respect to ratio # magenta vertices / size



PERSPECTIVES

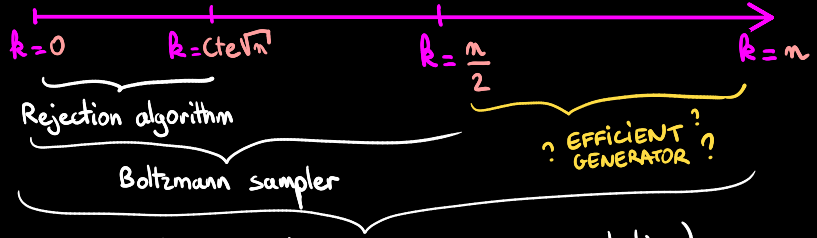
→ Asymptotic behaviour

- Asymptotic equivalent of # Git graphs of size n?
- Limit Laws
- Phase transition?

→

CONCLUSION

Sampling with respect to ratio # magenta vertices / size

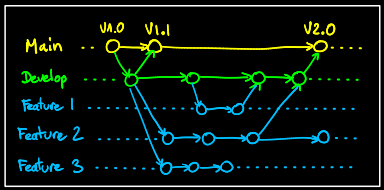


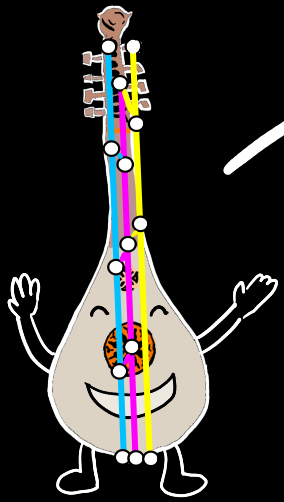
PERSPECTIVES

→ Asymptotic behaviour

- Asymptotic equivalent of # Git graphs of size n?
- Limit Laws
- Phase transition?

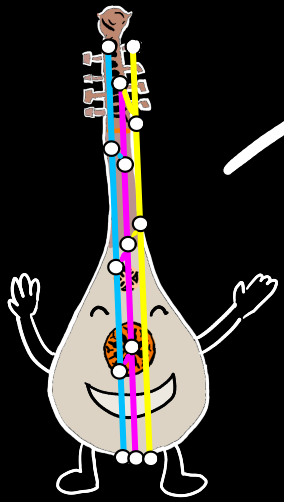
→ Other workflows?





THANK You!

Answer to the riddle :



THANK You!

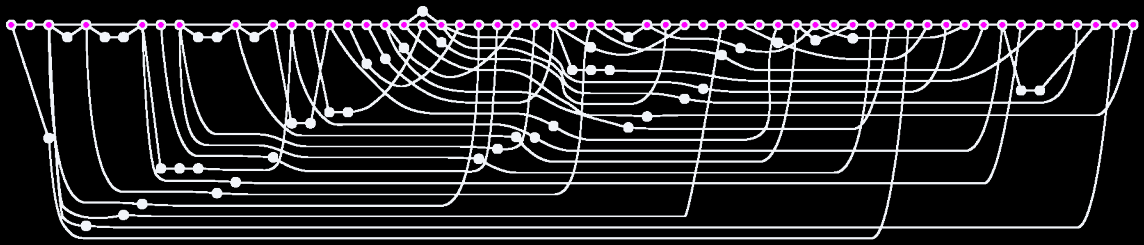
Answer to the riddle:
GITTERN

MOST GIT GRAPHS LOOK ALIKE

Why not just sampling given a size n ?

Theorem

In a Git graph of size n taken uniformly at random, the number of magenta vertices is $\frac{n}{2} + o(n)$



random Git graph of size 100

Sampling a Git graph of size n uniformly means aiming a number of magenta vertices $k \simeq \frac{n}{2}$