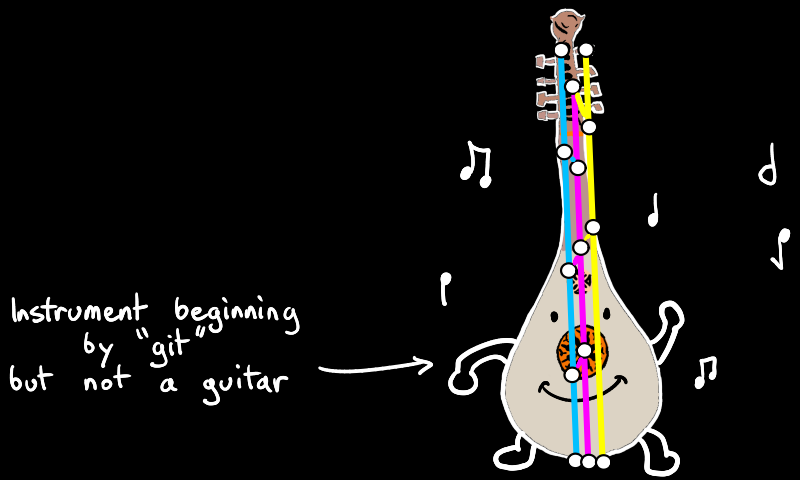
Random generation of Git graphs

Julien COURTIEL (Université de Caen Normandie)
with Martin PEPIN (Université de Caen Normandie)



QUESTION: What do you use to share files with your coauthors?

THE TIER LIST

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god-like

good

trash

spawn of the devil

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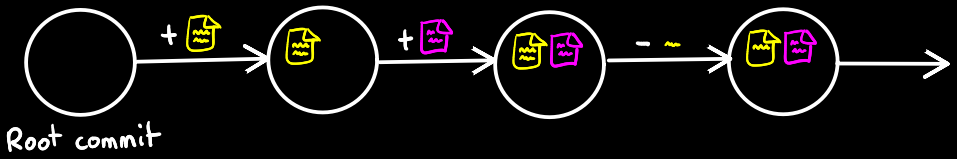
GIT FEATURES

git is a Version Control System (VCS): it stores all the project states over time.



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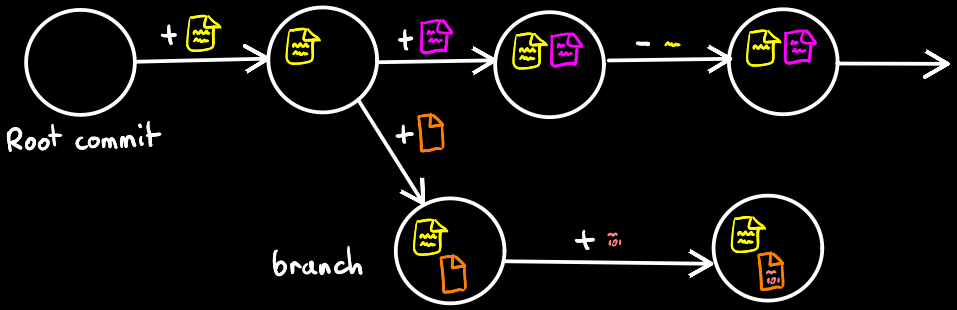

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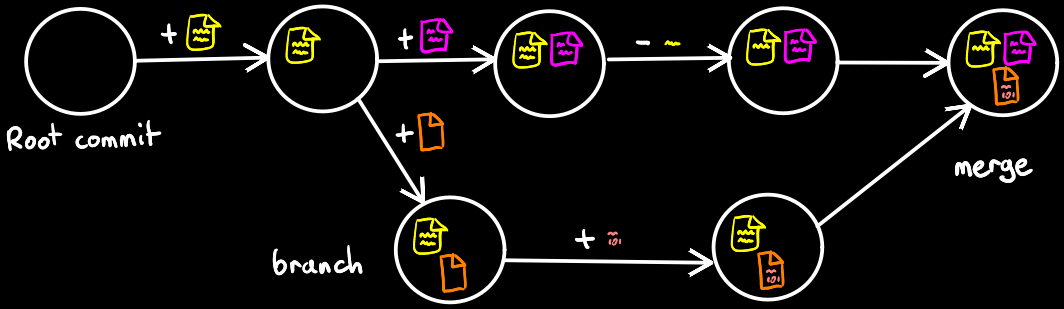

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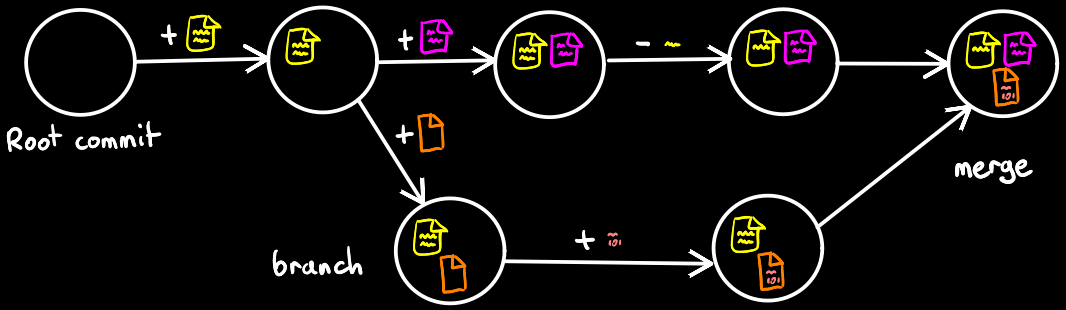

Git lets you create parallel development branches
that can be integrated later.

This forms a Directed Acyclic Graph (DAG),
where the vertices are the project states, also named commits.

OUR MOTIVATION

Objective : Design random generators for graphs of commits

Why?

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1. VCS unit testing
2. Benchmarks for VCS algorithms

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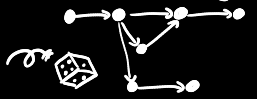
1. VCS unit testing

use random graphs



to check program correctness

```
void test_remove_last_commit() {

 srand(time(NULL));

 for (int i = 0; i < NUM_TESTS; i++) {

  GitGraph graph1 = create_random_git_graph();
  size_t n1 = graph1.num_commits;

  GitGraph graph2 = remove_last_commit(graph1);
  size_t n2 = graph2.num_commits;

  assert(n1 == n2 + 1);

 }
}
```

2. Benchmarks for VCS algorithms

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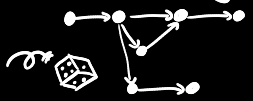

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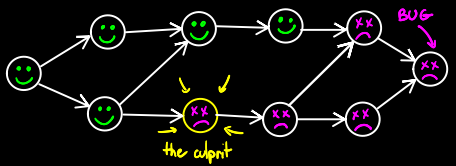
```

2. Benchmarks for VCS algorithms

e.g.

Regression Search Problem

Find the commit that introduces a bug

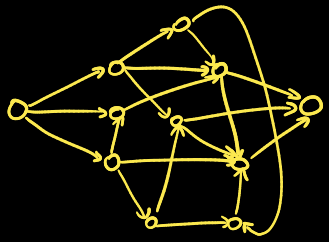


Analysis of algorithms such as git bisect

[C. Dorbec Lecoq 2023]

WHICH GRAPHS TO GENERATE?

In git, every DAG without restriction can be generated...



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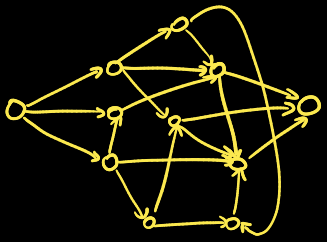
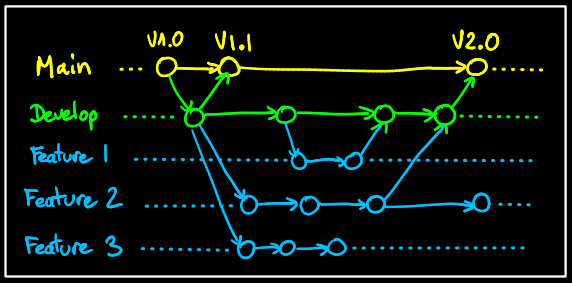


... but many projects follow a workflow



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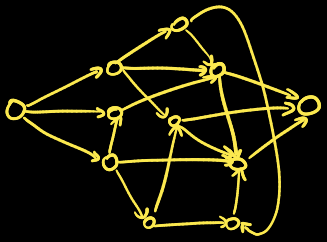

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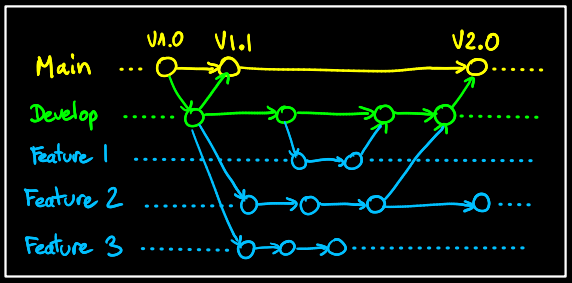


In this work, we consider a simple workflow but widely used in industry: the feature branch workflow

GIT GRAPH

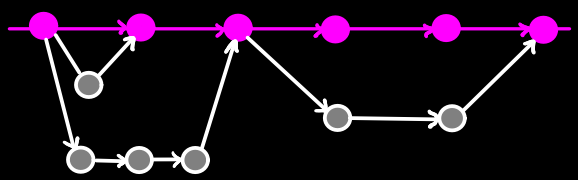
DEFINITION

(feature branch) Git graph = DAG with

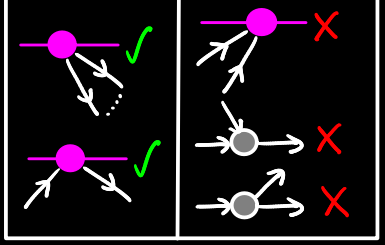
- a main branch (path of magenta vertices)
- 0, 1 or several feature branches, paths of ≥ 1 white vertices starting and ending on magenta vertices
- indegree ≤ 2 for all vertices

previously defined in [Lecoq 2024]

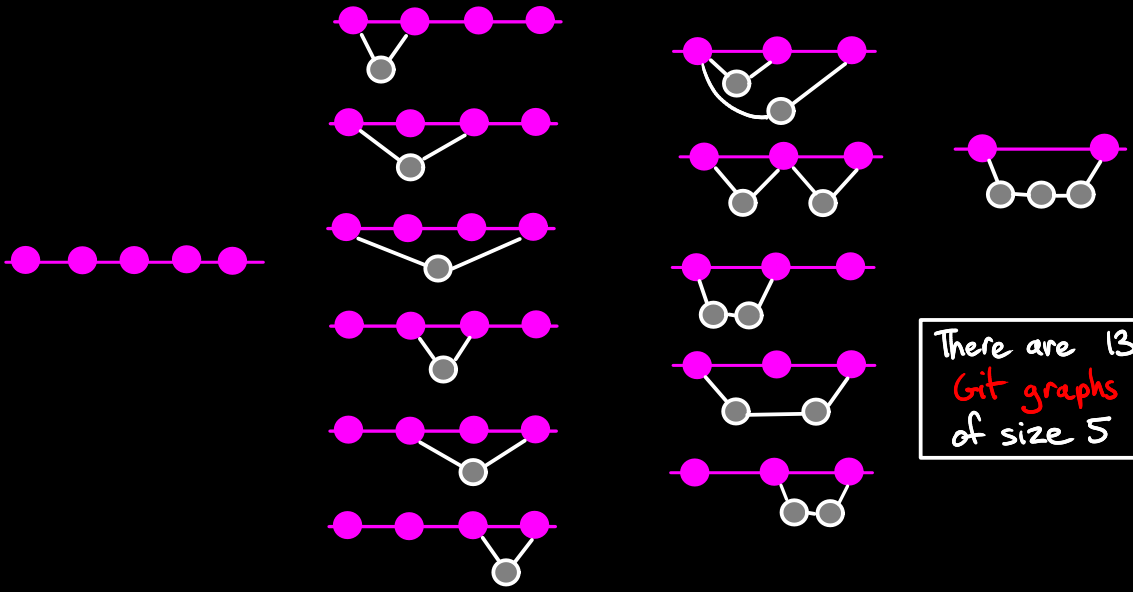
e.g.



ILLUSTRATED RULES



ENUMERATING & SAMPLING



There are 13 Git graphs of size 5

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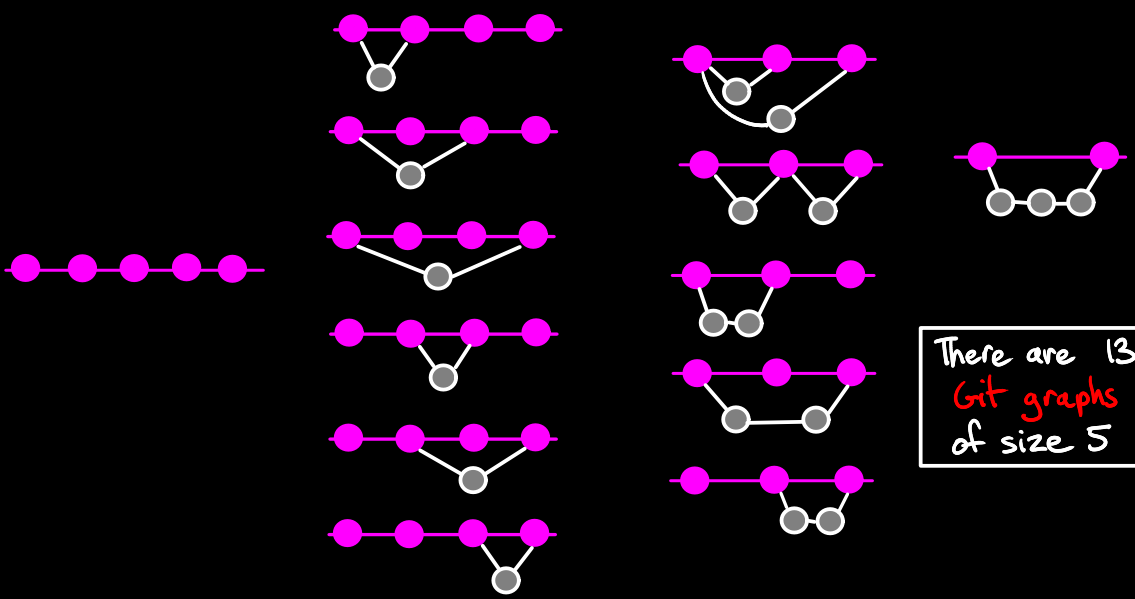


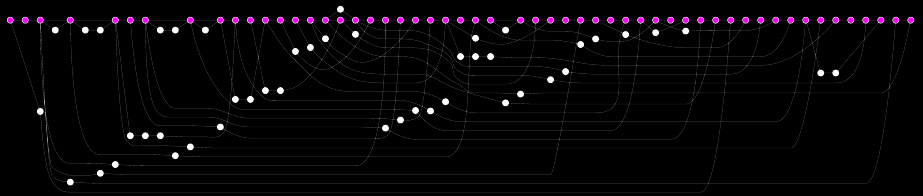
There are 13 Git graphs of size 5

GOAL	Sample a Git graph uniformly at random...	... given a size n NO ✗ not the most interesting	... given a size n and a number k of magenta vertices YES ✓

MOST GIT GRAPHS LOOK ALIKE

Theorem

In a Git graph of size n taken uniformly at random, the number of magenta vertices is $\frac{n}{2} + o(n)$ and the proportion of feature branches with exactly 1 white vertex tends to 1.



random Git graph of size 100

Proof: Bounding the number of Git graphs of size n with k magenta vertices

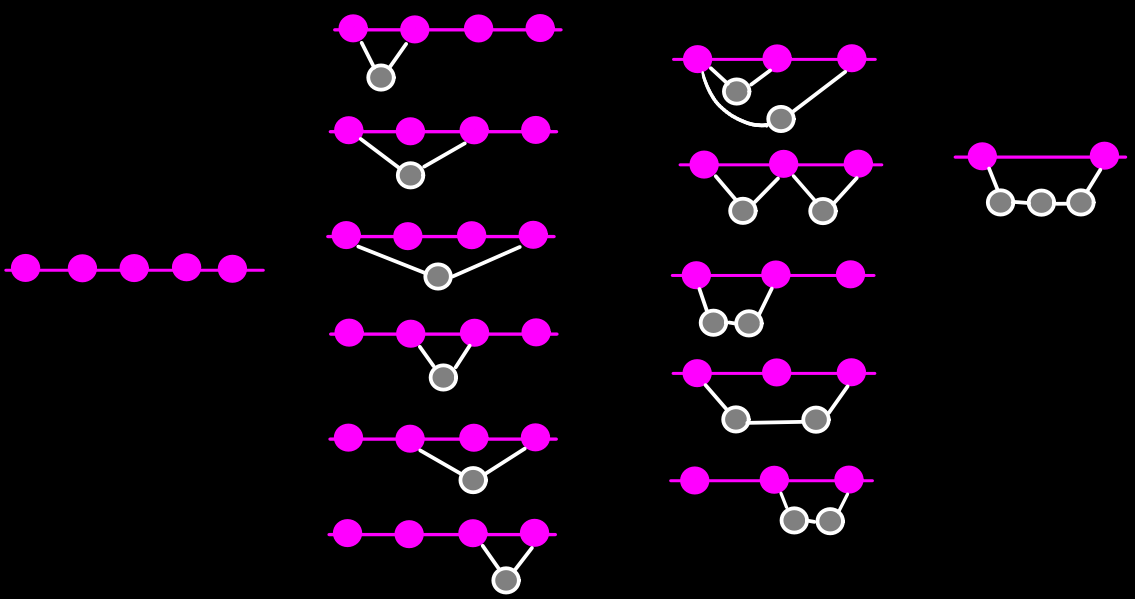
&

showing that only graphs such that

$$\left(\frac{1}{2} - \varepsilon\right)n \leq k \leq \left(\frac{1}{2} + \varepsilon\right)n$$

matter.

ENUMERATING & SAMPLING



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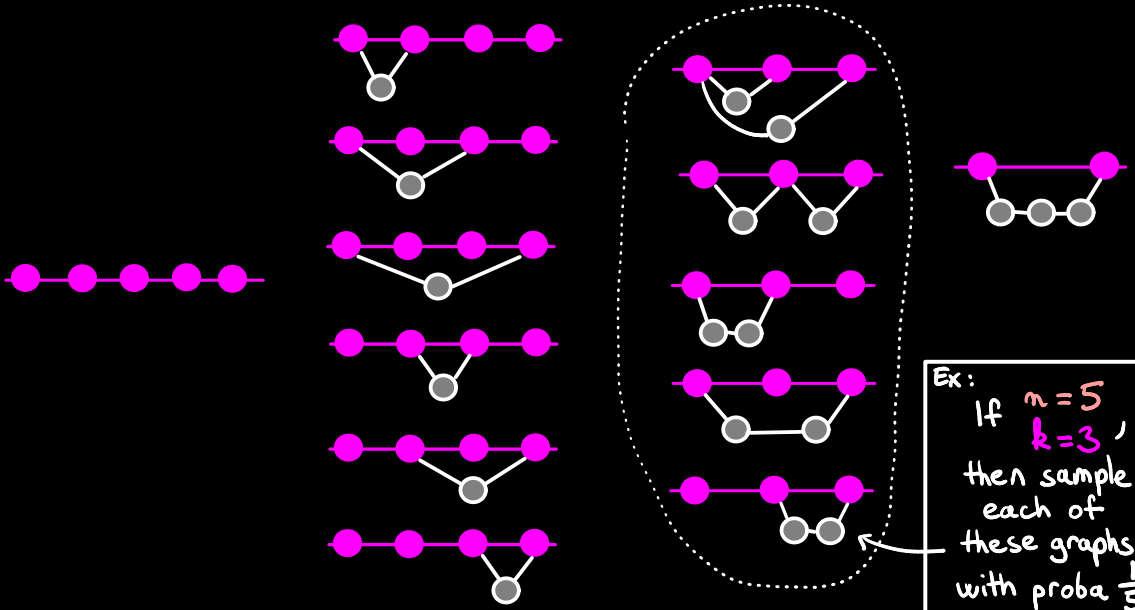


Ex:
If $n=5$, $k=3$, then sample each of these graphs with proba $\frac{1}{5}$

GOAL

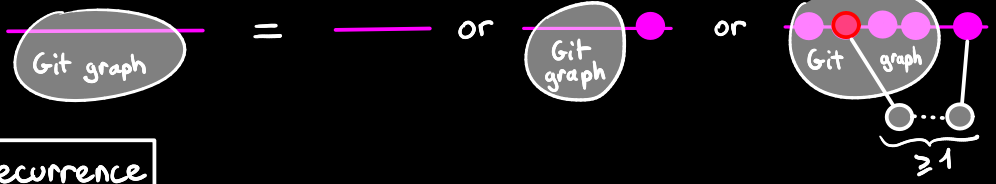
Sample a Git graph uniformly at random...

... given a size n NO ✗

... given a size n and a number k of magenta vertices YES ✓

RECURSIVE DECOMPOSITION

Decomposition



Recurrence

$$g_{n,k} = g_{n-1,k-1} + \sum_{\ell \geq 1} (k-1)\, g_{n-1-\ell,k-1} \qquad \text{for } n \geq 1$$

where $g_{n,k}$:= number of Git graphs with n vertices, k of them being magenta

Differential Equation for the Generating Function

$$G(z,u) = 1 + zu\,G(z,u) + \frac{z^2u^2}{1-z}\frac{\partial G}{\partial u}(z,u)$$

where $G(z,u) = \sum_{n \geq 0} \sum_{k \geq 0} g_{n,k}\, z^n u^k$

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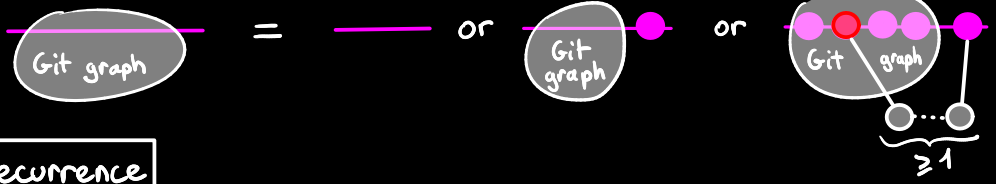

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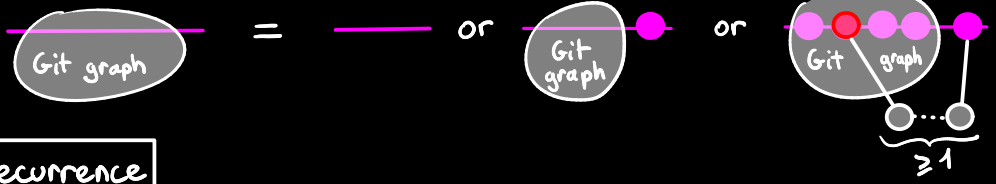

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→ possible to write a recursive generator from this but it is inefficient

→ Bolzmann sampling impossible (?) since $G(z,u)$ not analytic

TRANSFORMING THE EQUATION

Recurrence	$g_{n,k} = g_{n-1,k-1} + \sum_{l \geq 1} (k-1)\, g_{n-1-l,k-1}$
Differential Equation	$G(z,u) = 1 + zu\, G(z,u) + \frac{z^2 u^2}{1-z} \frac{\partial G}{\partial u}(z,u)$

Ordinary Generating Function

$$\underbrace{\sum_{n,k \geq 0} g_{n,k} z^n u^k}_{G(z,u),\ \text{not analytic}\ \times}$$

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Usual trick:

Ordinary Generating Function

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Borel transform

Exponential Generating Function

$$\underbrace{\sum_{n,k \geq 0} \frac{g_{n,k}}{n!} z^n u^k}_{\text{analytic}}$$

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$$\underbrace{\sum_{n,k \geq 0} \frac{g_{n,k}}{n!} z^n u^k}_{\text{analytic, but no pretty equation}\ \times}$$

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Usual trick:

Ordinary Generating Function $\sum_{n,k \geq 0} g_{n,k} z^n u^k$ $= G(z,u)$, not analytic ✗

Borel transform

Exponential Generating Function $\sum_{n,k \geq 0} \frac{g_{n,k}}{n!} z^n u^k$ analytic, but no pretty equation ✗

Borel transform on u

$$\tilde{G}(z,u) = \sum_{n,k \geq 0} \frac{g_{n,k}}{k!} z^n u^k$$

analytic ✓

and

Differential Equation for $\tilde{G}$
$\frac{\partial \tilde{G}}{\partial u} = z \tilde{G} + \frac{z^2 u}{1-z} \frac{\partial \tilde{G}}{\partial u}$ ✓

WHAT IS $\widetilde{G}(x,u)$?

Numbers: $g_{n,k}$ (counts Git graphs with respect to vertices/magenta vertices)

Generating Function:

$$\widetilde{G}(x,u) = \sum_{n,k\geq 0} \frac{g_{n,k}}{k!} x^n u^k$$

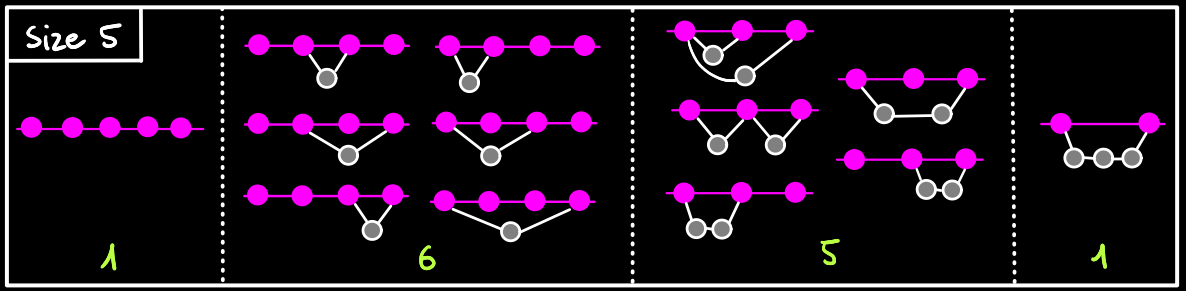
Differential Equation for $\widetilde{G}$

$$\frac{\partial \widetilde{G}}{\partial u} = x\widetilde{G} + \frac{x^2 u}{1-x}\frac{\partial \widetilde{G}}{\partial u}$$

First coefficients:

$$\widetilde{G}(x,u) = 1 + ux + \frac{u^2}{2!}x^2 + \left(\frac{u^3}{3!}+\frac{u^2}{2!}\right)x^3 + \left(\frac{u^4}{4!}+3\frac{u^3}{3!}+\frac{u^2}{2!}\right)x^4 + \left(\frac{u^5}{5!}+6\frac{u^4}{4!}+5\frac{u^3}{3!}+\frac{u^2}{2!}\right)x^5 + \dots$$

$\widetilde{G}$ is analytic.



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Theorem

$$\widetilde{G}(x,u) = \left(1 - \frac{x^2 u}{1-x}\right)^{-\frac{1-x}{x}}$$

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How can it be exploited?

RANDOM GENERATION?

RANDOM GENERATION?

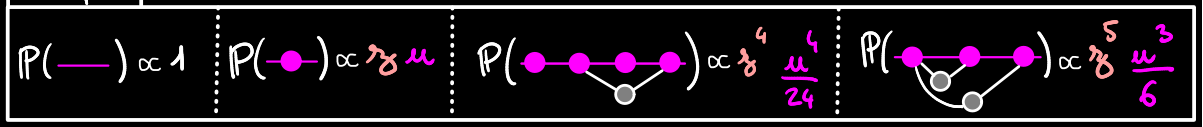
"Boltzmann model" (exponential in u, ordinary in z)

Fix $z > 0^*$ and $u > 0^*$.

We wish to sample a Git graph $\mathcal{G}$ with a weight proportional to $z^{\#\text{vertices in } \mathcal{G}} \dfrac{u^{\#\text{magenta vertices in } \mathcal{G}}}{(\#\text{magenta vertices in } \mathcal{G})!}$

(Size is not fixed)

Examples



*: in the disk of convergence of $\tilde{G}$

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Fix $x_\gamma > 0^*$ and $u > 0^*$.

We wish to sample a Git graph γ with a weight

~~proportional~~ equal to $\dfrac{x_\gamma^{\#\text{vertices in } \gamma}}{\widetilde{G}(x_\gamma, u)} \dfrac{u^{\#\text{magenta vertices in } \gamma}}{(\#\text{magenta vertices in } \gamma)!}$

(Size is not fixed)

where $\widetilde{G}(x_\gamma, u) = \sum_{m,k \geq 0} g_{m,k} \dfrac{x_\gamma^m u^k}{k!} = \left(1 - \dfrac{x_\gamma^2 u}{1 - x_\gamma}\right)^{-\frac{1-x_\gamma}{x_\gamma}}$.

Examples

$\mathbb{P}(\text{—}) = \dfrac{1}{\widetilde{G}(x_\gamma, u)}$ $\quad$ $\mathbb{P}(\text{—●—}) = \dfrac{x_\gamma u}{\widetilde{G}(x_\gamma, u)}$ $\quad$ $\mathbb{P}(\text{●—●—●—●}) = \dfrac{x_\gamma^4}{\widetilde{G}(x_\gamma, u)} \dfrac{u^4}{24}$ $\quad$ $\mathbb{P}(\text{●—●—●—●, ●—●}) = \dfrac{x_\gamma^5}{\widetilde{G}(x_\gamma, u)} \dfrac{u^3}{6}$

*: in the disk of convergence of $\widetilde{G}$

CHOOSING z AND u

Proposition

Let γ be a random Git Graph sampled with respect to the previous Boltzmann model, conditioned to have size n

$$\mathbb{E}(\#\text{magenta vertices}(\gamma)) \sim \frac{1-\rho_u}{2-\rho_u} n$$

$$\mathbb{V}(\#\text{magenta vertices}(\gamma)) \sim \frac{\rho_u(1-\rho_u)}{(2-\rho_u)^3} n$$

where $\rho_u = \frac{\sqrt{1+4u}-1}{2u}$

Proof : Transfer Theorem from $\tilde{G}(z_\gamma, u) = \left(1 - \frac{z_\gamma^2 u}{1-z_\gamma}\right)^{-\frac{1-z_\gamma}{z_\gamma}}$

CHOOSING z_γ AND u

Proposition

Let γ be a random Git Graph sampled with respect to the previous Boltzmann model, conditioned to have size n

$$\mathbb{E}(\#\text{magenta vertices}(\gamma)) \sim \frac{1-\rho_u}{2-\rho_u} n$$

$$\mathbb{V}(\#\text{magenta vertices}(\gamma)) \sim \frac{\rho_u(1-\rho_u)}{(2-\rho_u)^3} n$$

where $\rho_u = \frac{\sqrt{1+4u}-1}{2u}$

Proof: Transfer Theorem from $\widetilde{G}(z_\gamma, u) = \left(1 - \frac{z_\gamma^2 u}{1-z_\gamma}\right)^{-\frac{1-z_\gamma}{z_\gamma}}$

Consequence: Given any $\alpha \in (0, \frac{1}{2})$, and $n \geq 0$, we can tune u to target αn magenta vertices and then z_γ to target size n.

Different Perspective

Is there a combinatorial explanation for the formula

$$\widetilde{G}(x_\delta, u) = \left(1 - \frac{x_\delta^2 u}{1 - x_\delta}\right)^{-\frac{1-x_\delta}{x_\delta}} ?$$

Different Perspective

Is there a combinatorial explanation for the formula

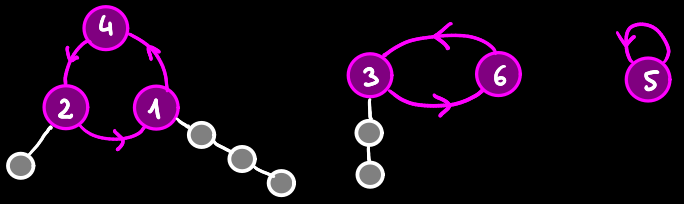
$$\widetilde{G}(x_\gamma, u) = \left(1 - \frac{x_\gamma^2 u}{1 - x_\gamma}\right)^{-\frac{1-x_\gamma}{x_\gamma}} = \exp\left(\frac{1}{\frac{x_\gamma}{1-x_\gamma}} \ln\left(\frac{1}{1 - u x_\gamma \frac{x_\gamma}{1-x_\gamma}}\right)\right)?$$

DIFFERENT PERSPECTIVE

Definition

cyclarium = set of cycles of magenta vertices labeled from 1 to k where a chain of white unlabeled vertices is attached to each magenta vertex, except to the ones having the largest label in their cycles.

e.g.:



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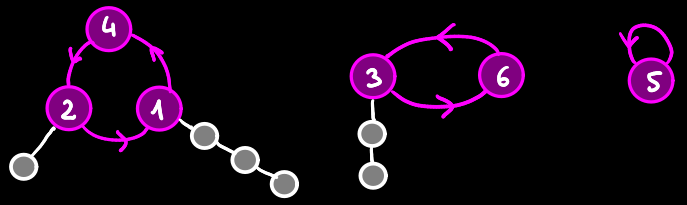

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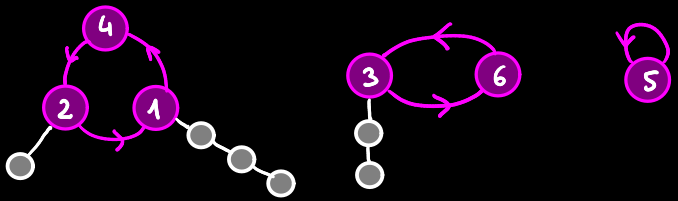
It's the generating function of cyclariums!

BIJECTION

Proposition There is a bijection from cyclariums to Git graphs.



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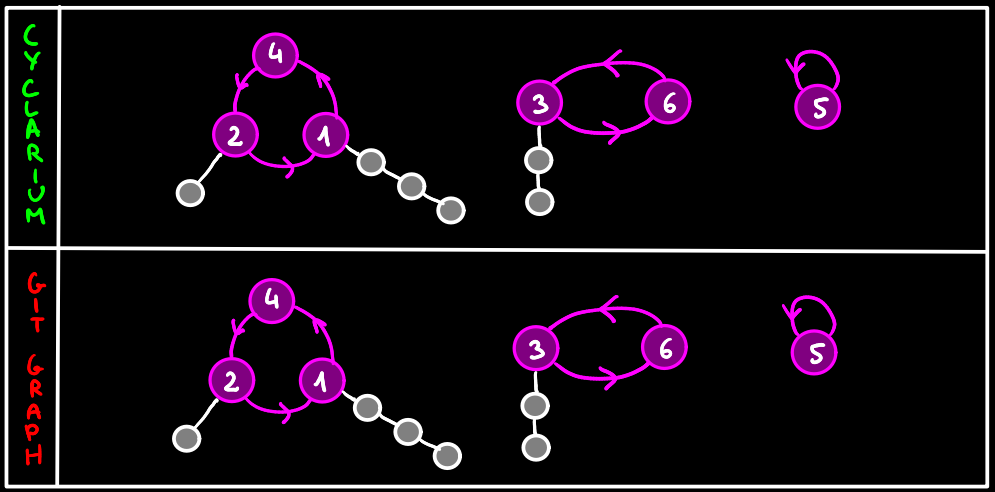

BIJECTION

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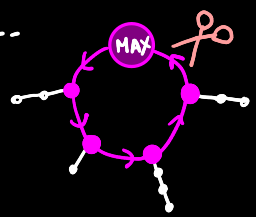
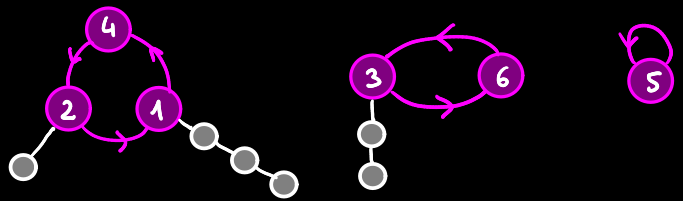
There is a bijection from cyclariums to Git graphs:

STEPS

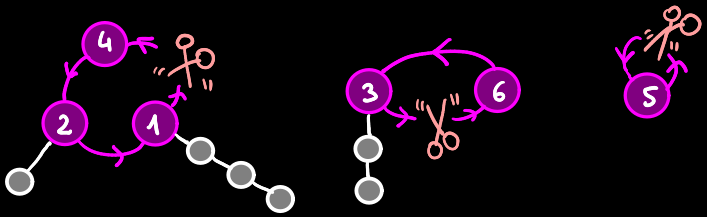
1-



CYCLARIUM



GIT GRAPH



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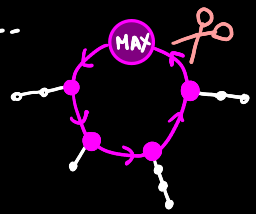

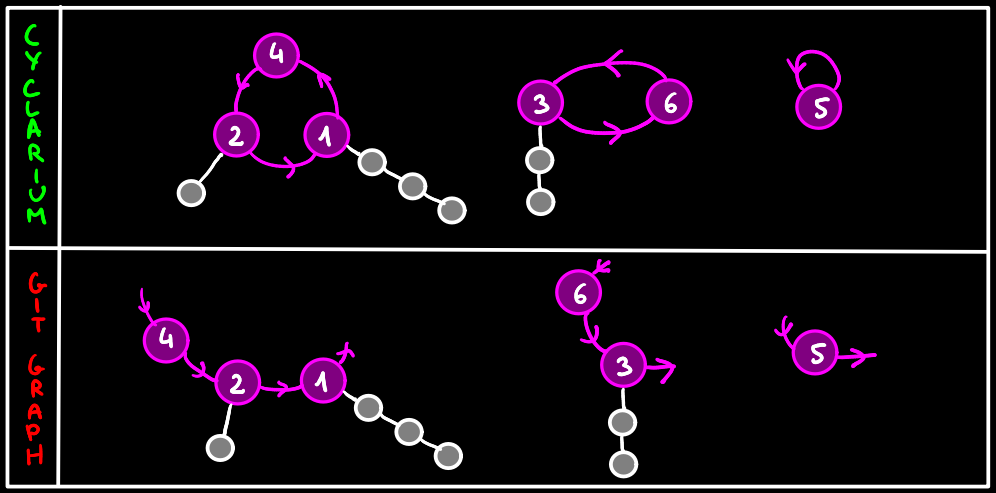

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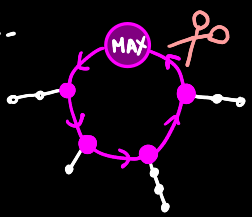

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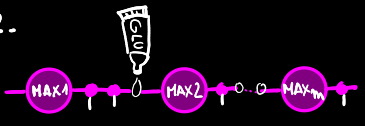

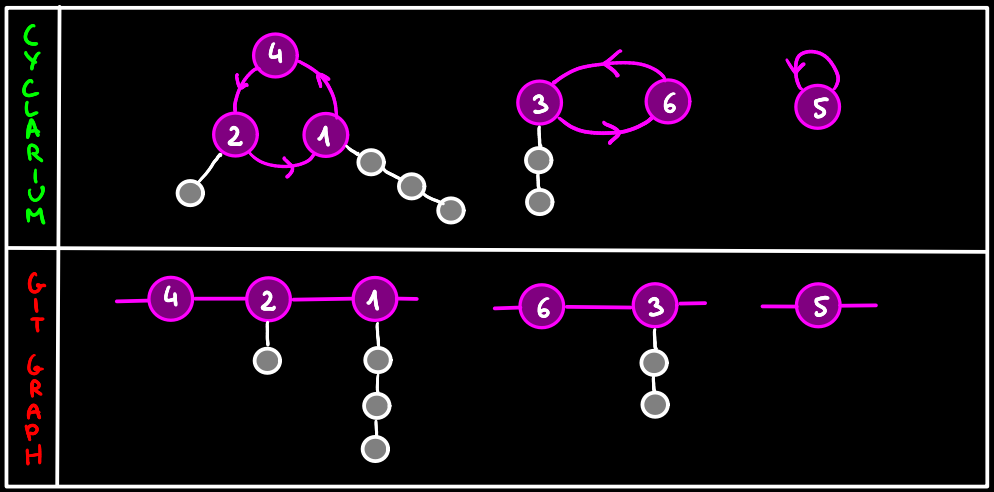

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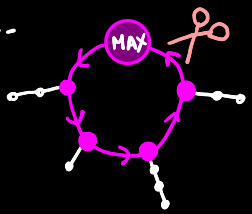
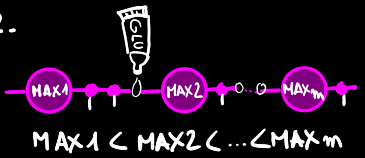
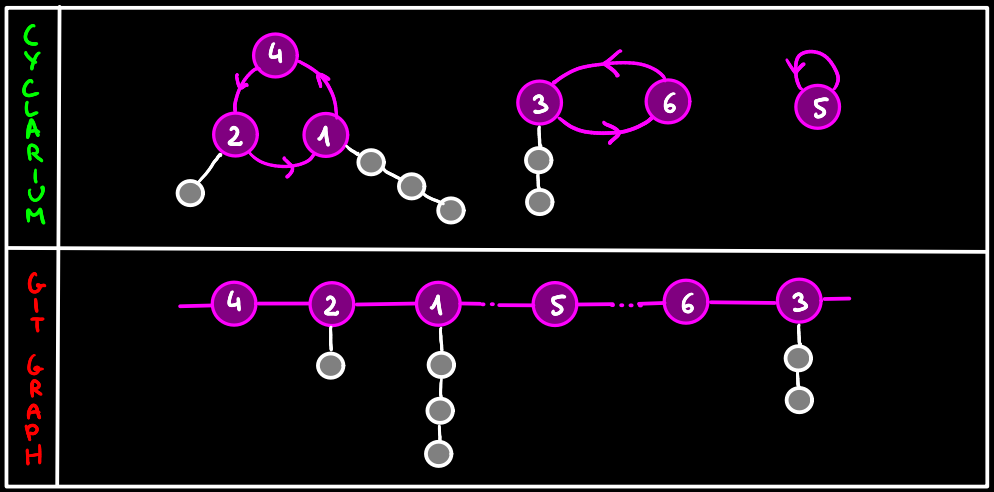


2.





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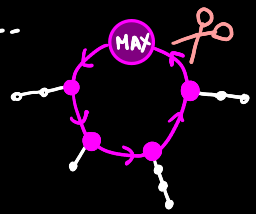

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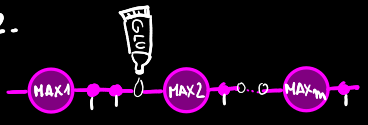
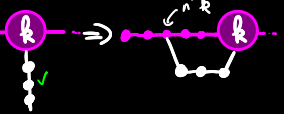
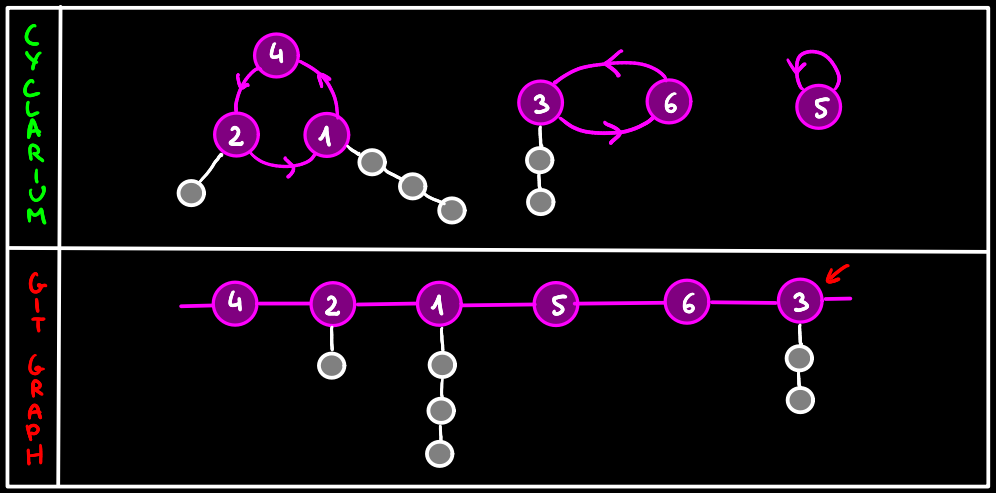


3. Right to left:

3a.



3b. Label removing + shift



BIJECTION

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There is a bijection from cyclariums to Git graphs:

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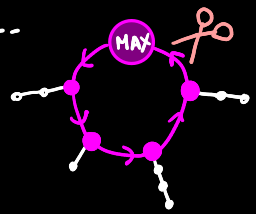

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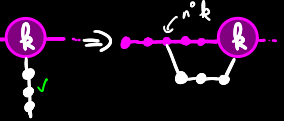

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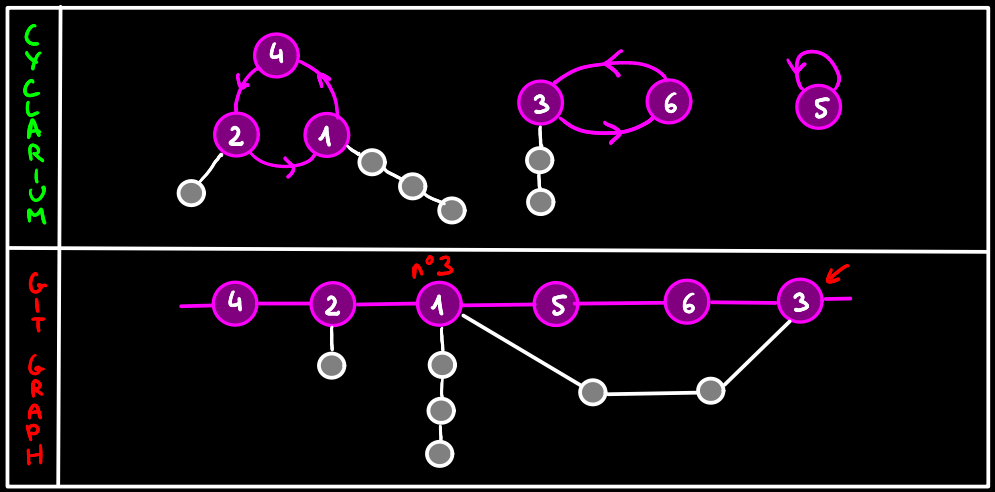

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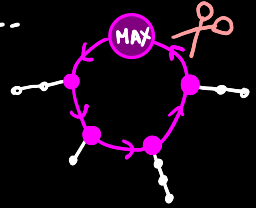

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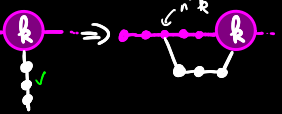

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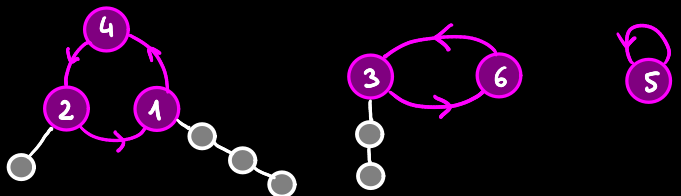
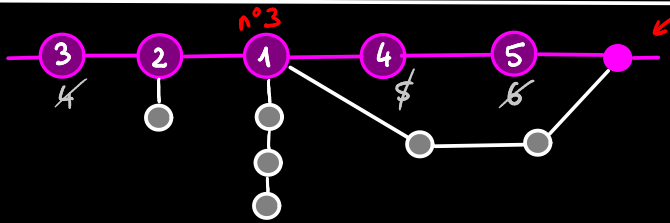
CYCLARIUM



GIT GRAPH



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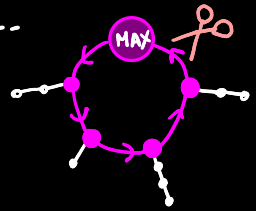

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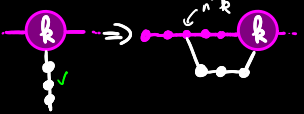

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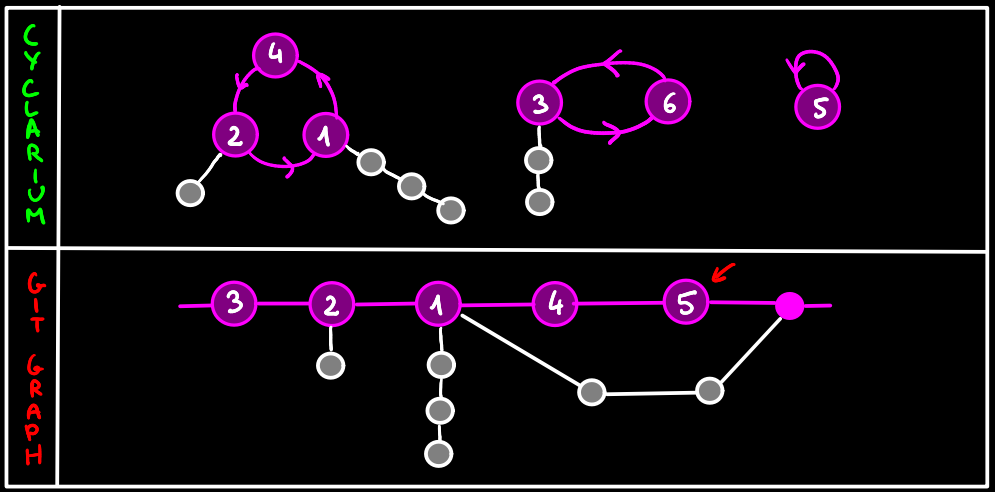

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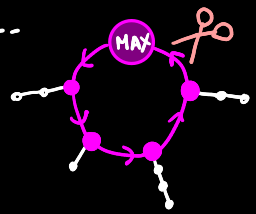

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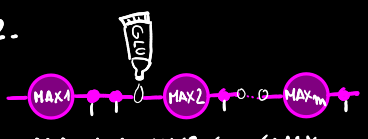

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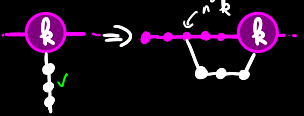
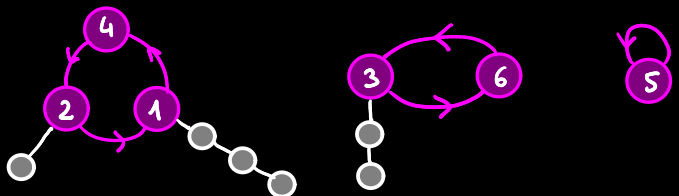
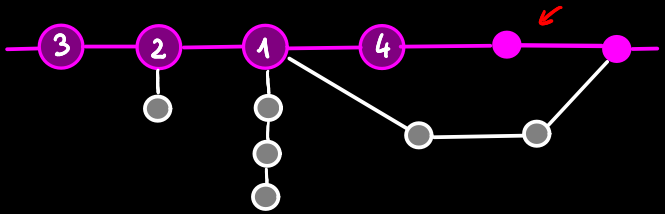


3b. Label removing + shift

CYCLARIUM



GIT GRAPH



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There is a bijection from cyclariums to Git graphs:

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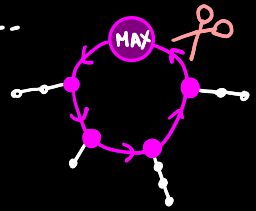

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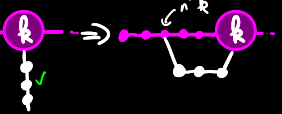

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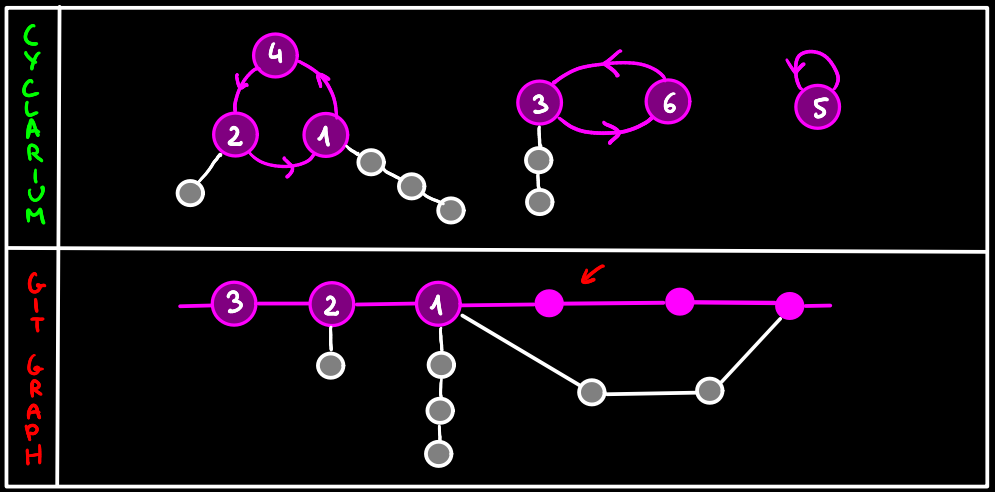

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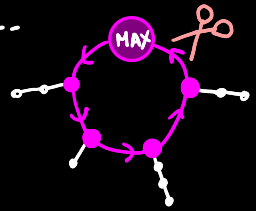

2.

MAX1 < MAX2 < ... < MAXm

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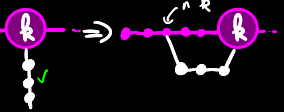

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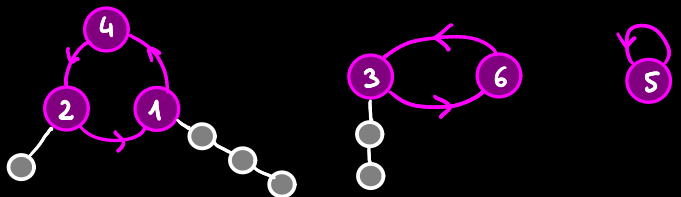
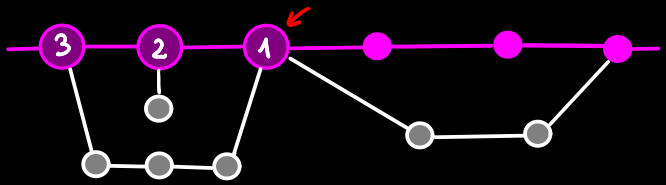


GIT GRAPH



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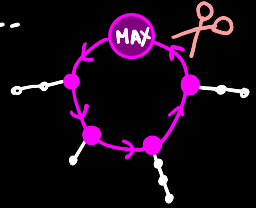

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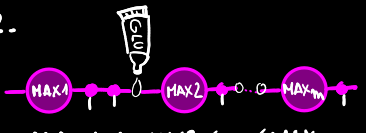

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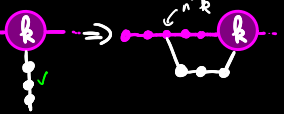

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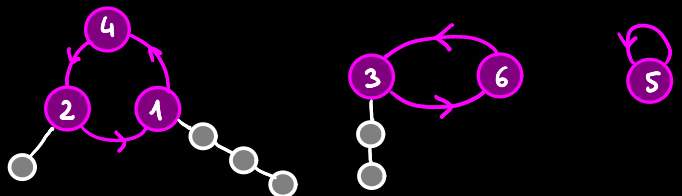
CYCLARIUM



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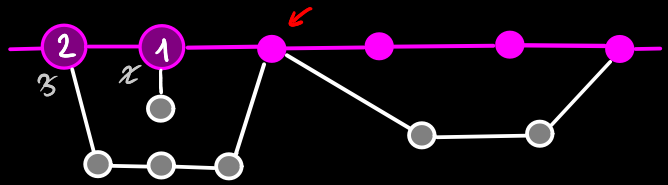

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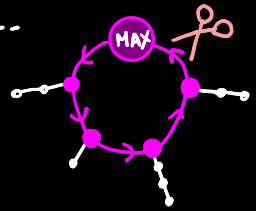

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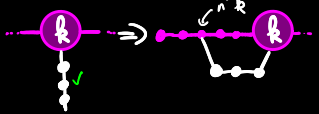

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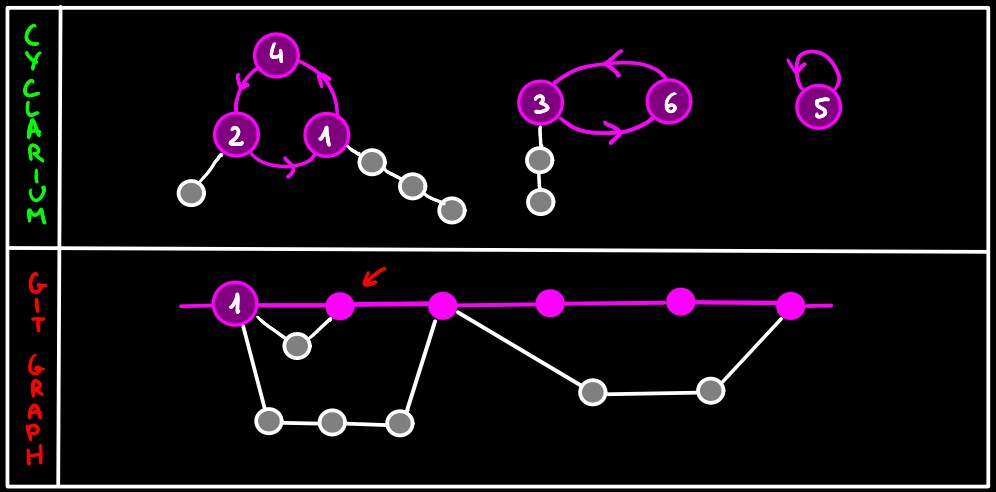

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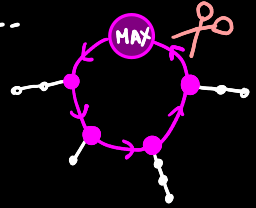
STEPS

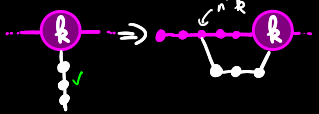
1.



2.

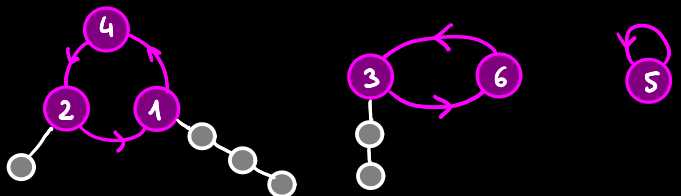
3. Right to left:

3a.

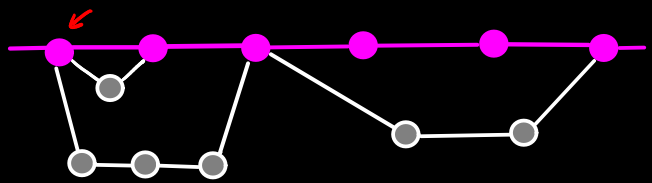


3b. Label removing + shift

CYCLARIUM



GIT GRAPH



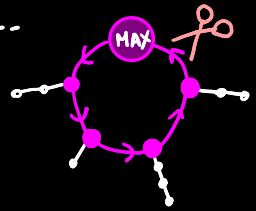
BIJECTION

Proposition

There is a bijection from cyclariums to Git graphs:

STEPS

1.



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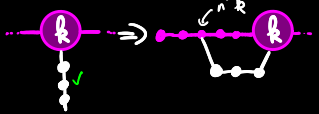

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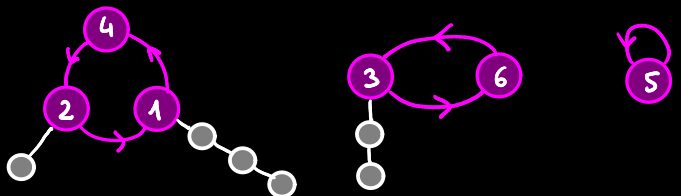
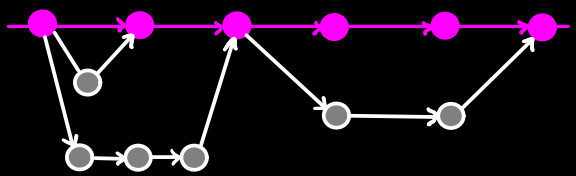


GIT GRAPH



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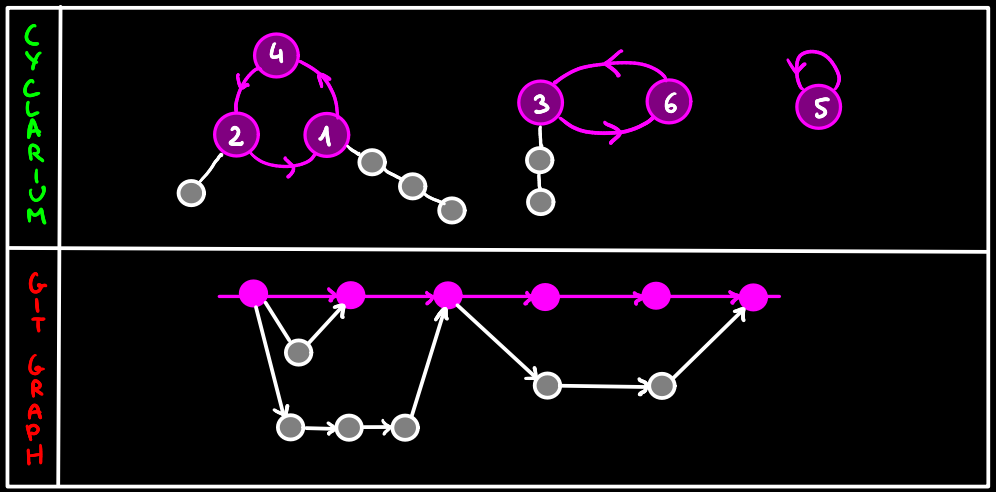
There is a bijection from cyclariums to Git graphs:
sending
vertices ⟶ vertices
magenta vertices ⟶ magenta vertices



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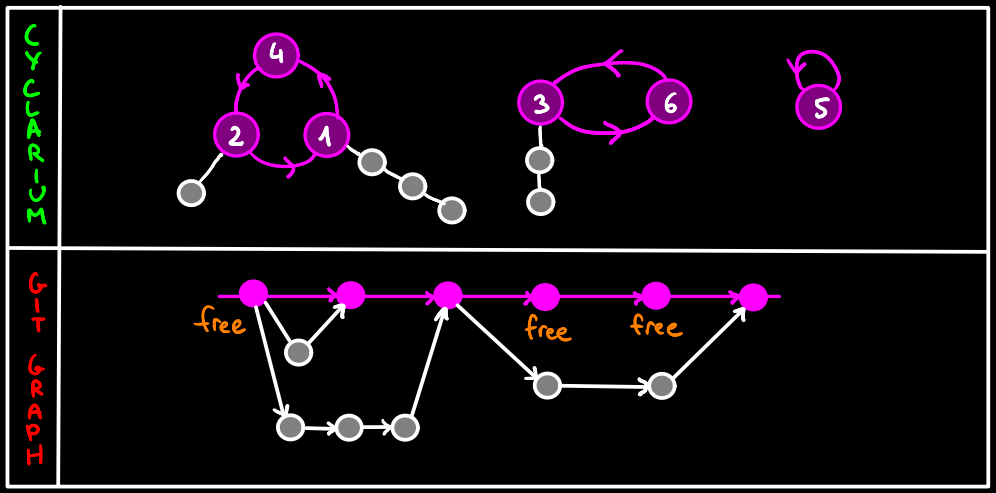
cycles ⟶ free vertices
i.e magenta vertices of indegree ≤ 1

cycle lengths ⟶ gaps between free vertices



A NICE FORMULA

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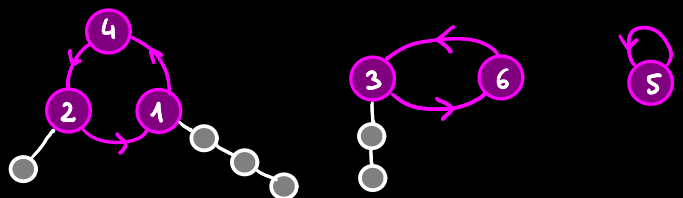
CYCLARIUM



Corollary

$$g_{n,k} = \sum_{f=1}^{k-1} \begin{bmatrix} k \\ f \end{bmatrix} \binom{n-k-1}{k-f-1} \qquad (k < n)$$

where $g_{n,k}$ = number of Git graphs counted by vertices & magenta vertices
and $\begin{bmatrix} : \\ : \end{bmatrix}$ = (unsigned) Stirling number of 1st kind

A UNIFORM SAMPLER

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RANDOM GENERATOR

Input : size n + # magenta vertices k + # free vertices f

- Generate a random permutation of size k with f cycles
- Randomly distribute $n-k$ white vertices into $k-f$ chains
- Form a cyclarium
- Use the bijection to get a Git graph

Complexity : $O(n)$

A UNIFORM SAMPLER

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Complexity : $O(n)$

But... precomputation of $O(f \times (k-f))$ large numbers

Remark: k and f can be sampled

BOLTZMANN SAMPLER

Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

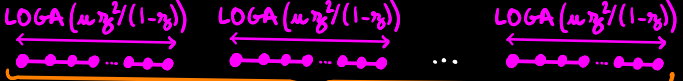
Output: A Git graph $\mathcal{G}$ with proba $z_g^{\text{size}(\mathcal{G})}\, u^{\#\text{magenta}(\mathcal{G})} / (\#\text{magenta}(\mathcal{G})!\, \widetilde{G}(z_g, u))$

BOLTZMANN SAMPLER

Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph γ with proba $z_g^{\text{size}(\gamma)} u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \tilde{G}(z_g, u))$

Step 1. Sample



Ex:

POISSON = 3

BOLTZMANN SAMPLER

Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph γ with proba $z_g^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z_g, u))$

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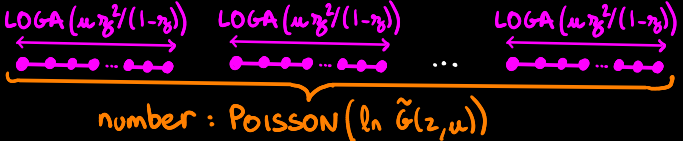
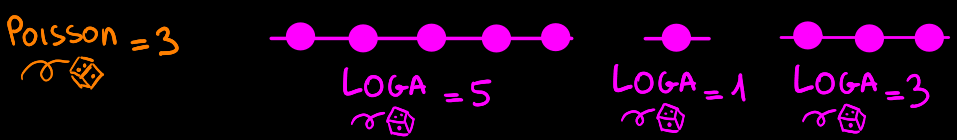


Ex:



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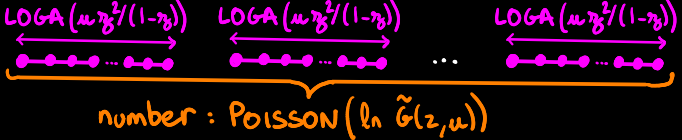
Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

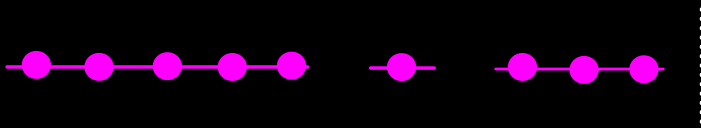
Output: A Git graph γ with proba $z_g^{\text{size}(\gamma)} u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z_g, u))$

Step 1. Sample



Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Ex:

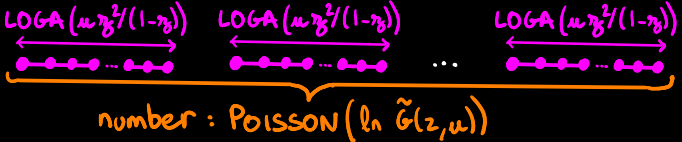


BOLTZMANN SAMPLER

Input $x_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph g with proba $x_g^{\text{size}(g)}\, u^{\#\text{magenta}(g)} / (\#\text{magenta}(g)!\ \widetilde{G}(x_g, u))$

Step 1. Sample



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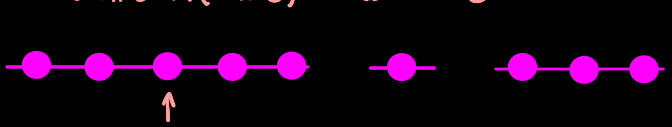
Ex:

UNIFORM $(1 \ldots 9)$ = 3

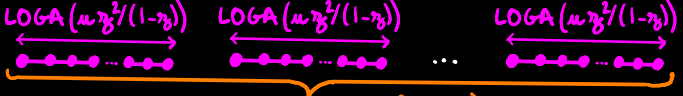


BOLTZMANN SAMPLER

Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph γ with proba $z_g^{\text{size}(\gamma)} u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z_g, u))$

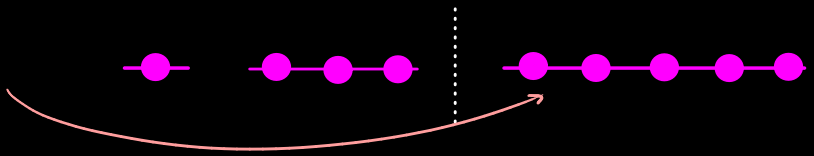
Step 1. Sample



Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

S

Ex:



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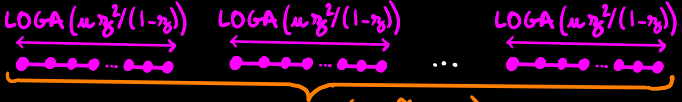

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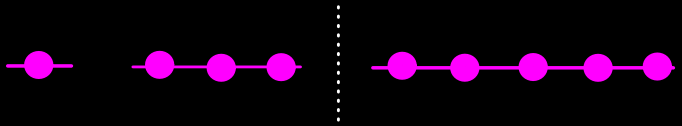
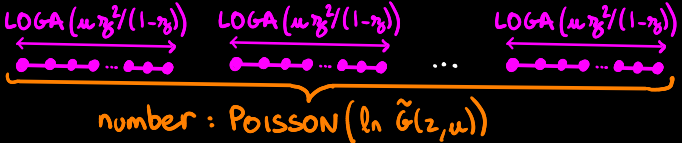


BOLTZMANN SAMPLER

Input $z_\gamma > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

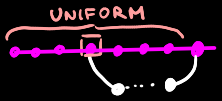
Output : A Git graph γ with proba $z_\gamma^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z_\gamma, u))$

Step 1. Sample



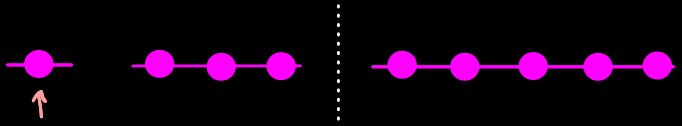
Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Step 3.  $1 + \text{GEOM}(z_\gamma)$ for 

Step 4. For each  , 

Ex :

UNIFORM $(1 \ldots 4)$  $= 1$



BOLTZMANN SAMPLER

Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

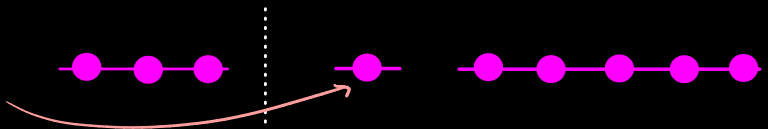
Output: A Git graph $\mathcal{G}$ with proba $z_g^{\text{size}(\mathcal{G})}\, u^{\#\text{magenta}(\mathcal{G})} / (\#\text{magenta}(\mathcal{G})!\, \widetilde{G}(z_g, u))$

Step 1. Sample

Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Ex:

UNIFORM(1...4) = 1



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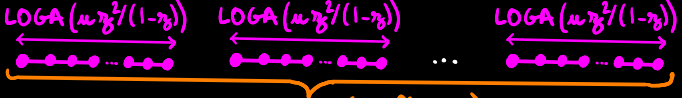
Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output: A Git graph γ with proba $z_g^{\text{size}(\gamma)} u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z_g, u))$

Step 1. Sample



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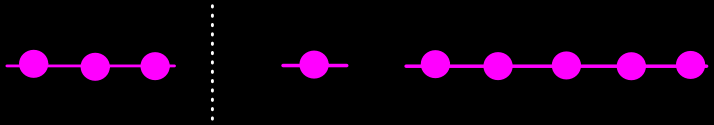

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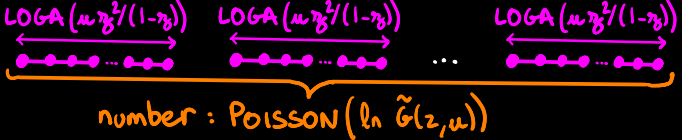

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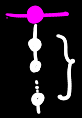
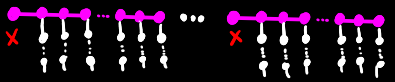
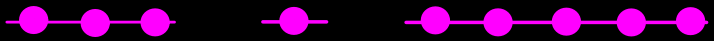
Step 3.  $1 + \text{GEOM}(z_g)$ for 

Ex: 

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Input $z_\gamma > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

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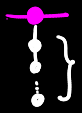
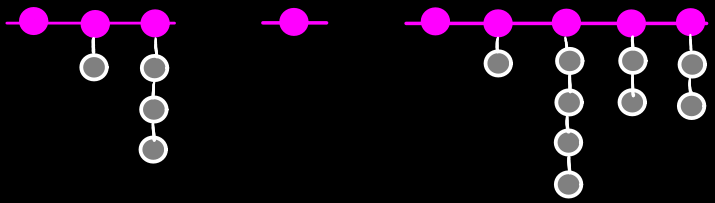
Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Step 3.  $1 + \text{GEOM}(z_\gamma)$ for 

Ex:

GEOM  x6



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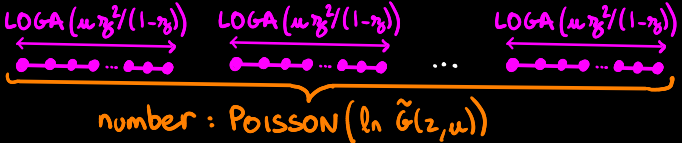

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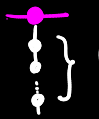
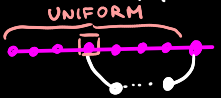
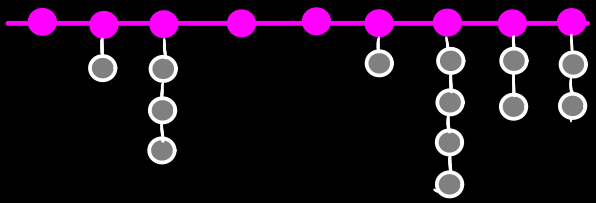
Step 3.  $1 + \text{GEOM}(z_g)$ for 

Step 4. Glue + for each  , 

Ex:



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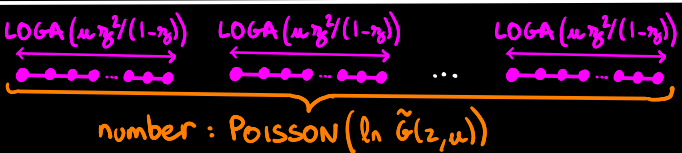
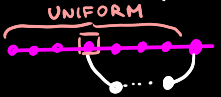
Output: A Git graph γ with proba $z_g^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z_g, u))$

Step 1. Sample

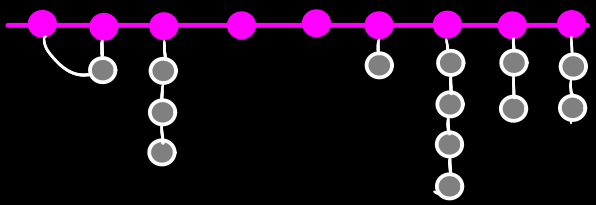


Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Step 3.  $1 + \text{GEOM}(z_g)$ for 

Step 4. Glue + for each  , 

Ex:

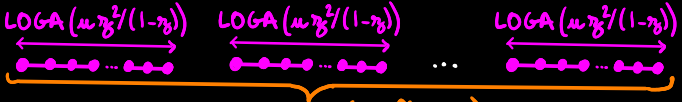


BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph δ with proba $z^{\text{size}(\delta)} u^{\#\text{magenta}(\delta)} / (\#\text{magenta}(\delta)!\ \tilde{G}(z,u))$

Step 1. Sample



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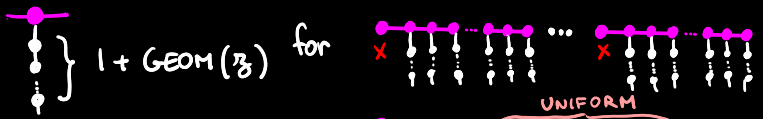
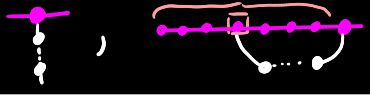
Step 3. 

Step 4. Glue + for each 

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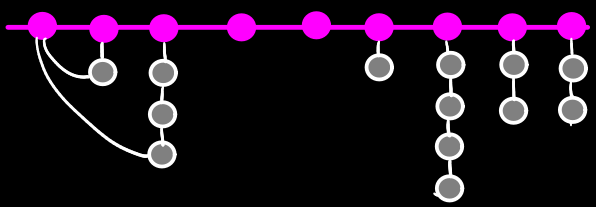
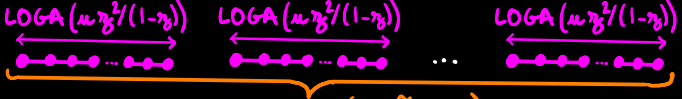


BOLTZMANN SAMPLER

Input $z > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

Output : A Git graph γ with proba $z^{\text{size}(\gamma)}\, u^{\#\text{magenta}(\gamma)} / (\#\text{magenta}(\gamma)!\, \widetilde{G}(z,u))$

Step 1. Sample



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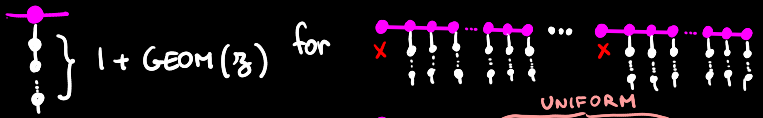
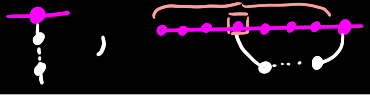
Step 3. 

Step 4. Glue + for each 

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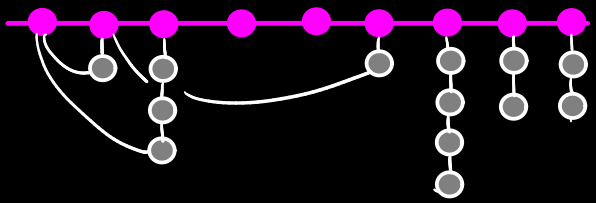
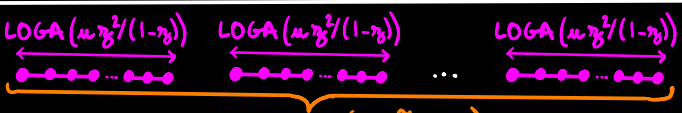


BOLTZMANN SAMPLER

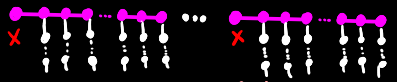
Input $z_g > 0$ and $u > 0$, tuned to target a size n & a # magenta vertices k

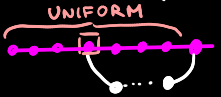
Output : A Git graph $\mathcal{G}$ with proba $z_g^{\text{size}(\mathcal{G})}\, u^{\#\text{magenta}(\mathcal{G})} / (\#\text{magenta}(\mathcal{G})!\, \widetilde{G}(z_g, u))$

Step 1. Sample

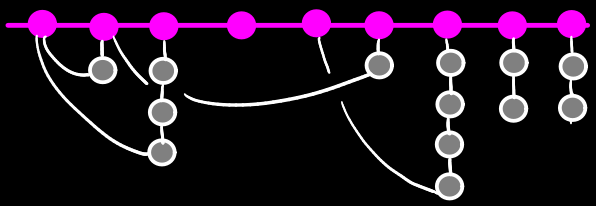


Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Step 3.  $1 + \text{GEOM}(z_g)$ for 

Step 4. Glue + for each  , 

Ex:



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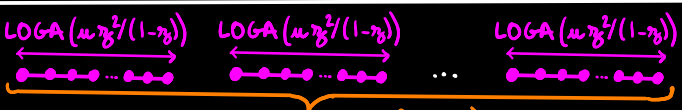
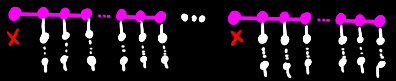
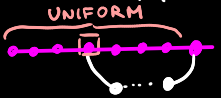


Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Step 3.  $1 + \text{GEOM}(z)$ for 

Step 4. Glue + for each  , 

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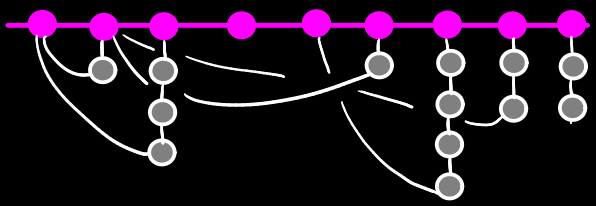

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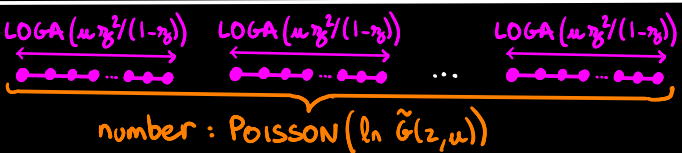
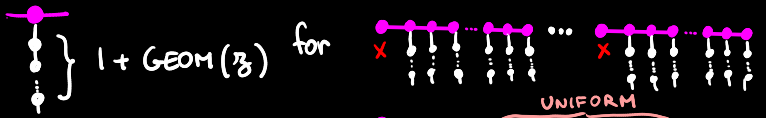
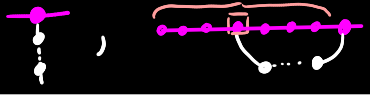
Step 1. Sample



Step 2. Biased shuffle (Pick a • unif. at random & put the segment on the right & repeat)

Step 3. 

Step 4. Glue + for each 

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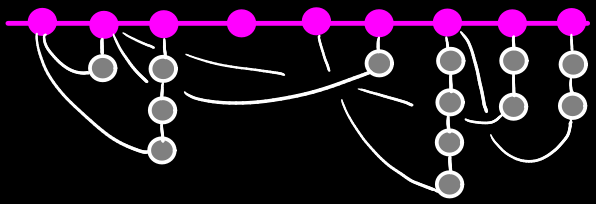
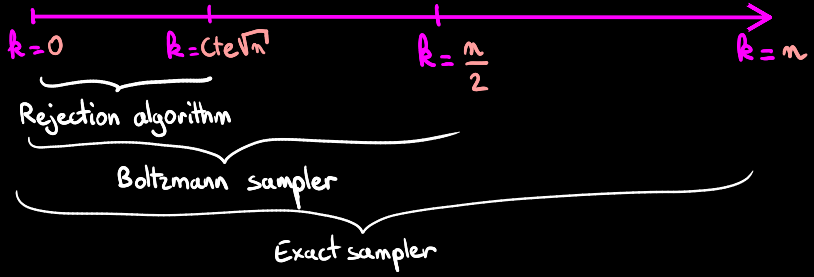


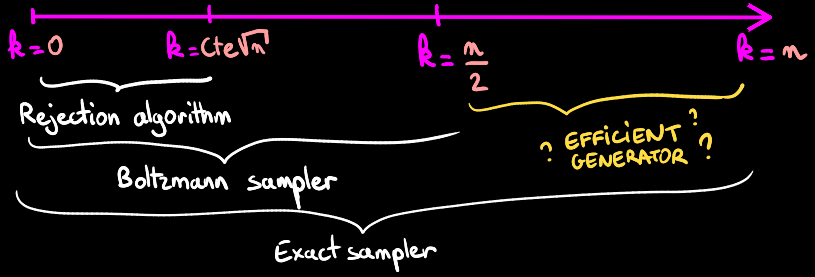
CONCLUSION

Sampling with respect to ratio # magenta vertices / size



CONCLUSION

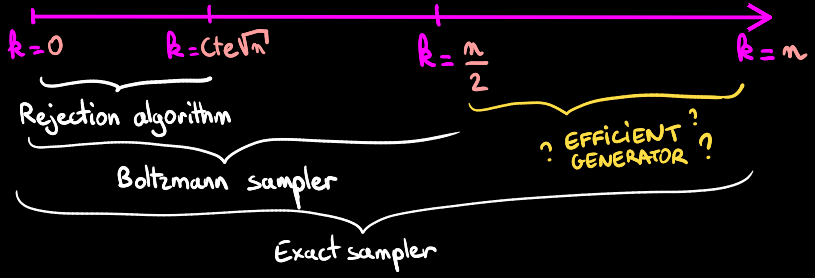
Sampling with respect to ratio # magenta vertices / size



PERSPECTIVES

CONCLUSION

Sampling with respect to ratio $\frac{\#\text{ magenta vertices}}{\text{size}}$

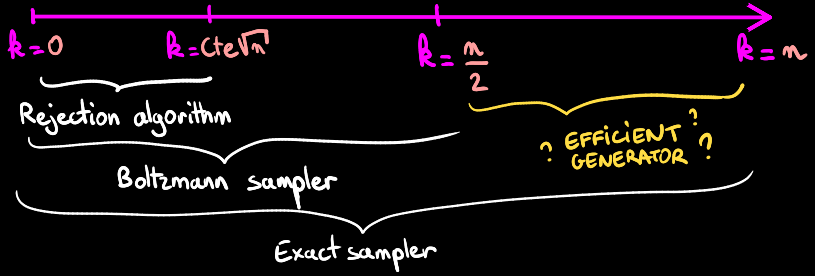


PERSPECTIVES → Asymptotic behaviour

- Asymptotic equivalent of # Git graphs of size n?
- Phase transition ?

CONCLUSION

Sampling with respect to ratio $\frac{\#\text{ magenta vertices}}{\text{size}}$

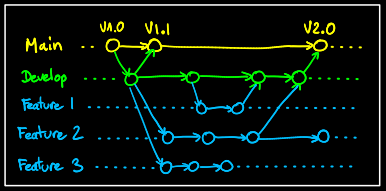


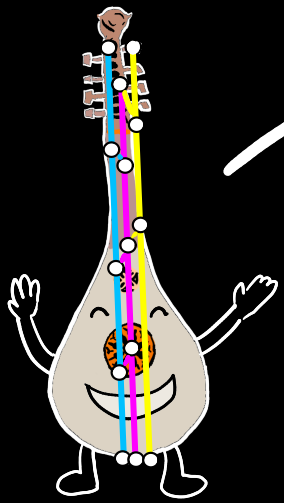
PERSPECTIVES

→ Asymptotic behaviour

- Asymptotic equivalent of # Git graphs of size n?
- Phase transition?

→ Other workflows?





THANK You!