COMBINATORICS OF GIT GRAPHS

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(Université de Caen Normandie)

WORK 1
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WORK 2
with Martin PEPIN (Université de Caen Normandie)

LIGM Seminar January 28th 2025

TIER LIST MADE IN MARNE

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QUESTION: What do you use to share files with your coauthors?

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god-like

good

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<u>QUESTION</u>: What do you use to share files with your coauthors?

god-like	git	mercurial
good	SUBVERSION	
trash		Overleaf
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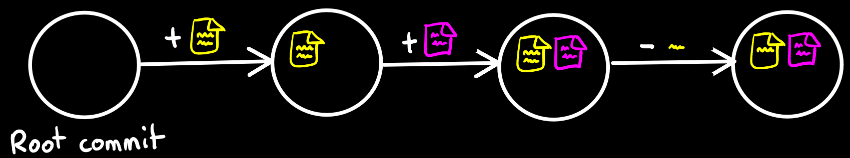
GIT FEATURES

git is a Version Control System (VCS) :
it stores all the project states over time.



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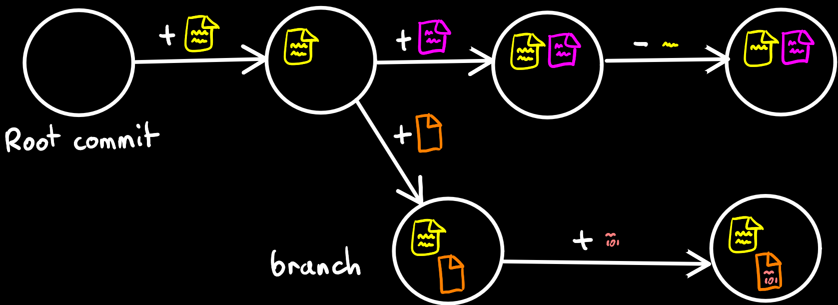

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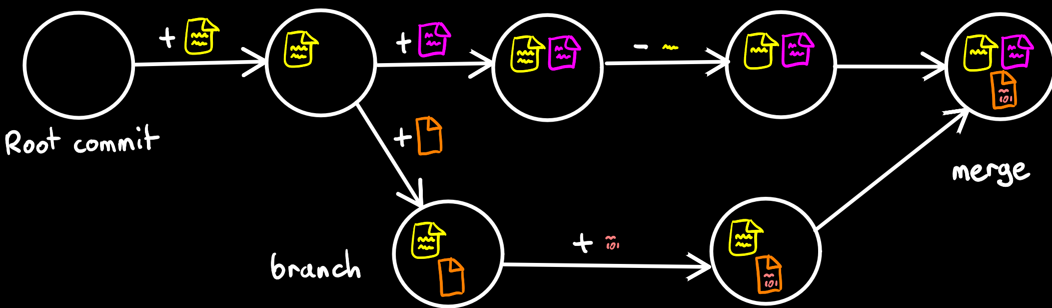

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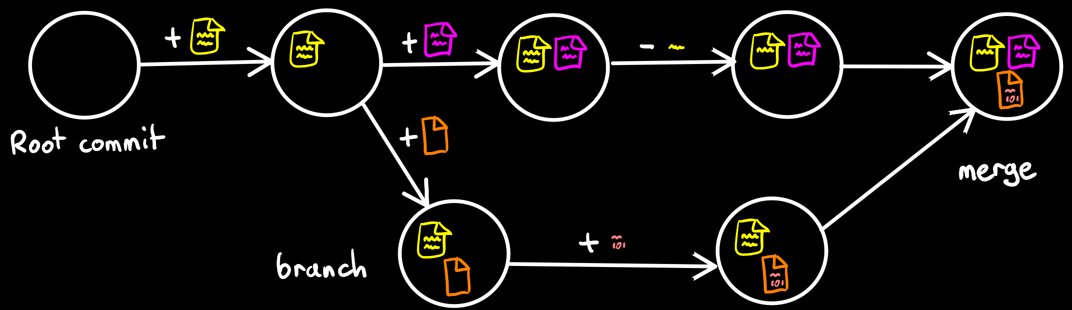

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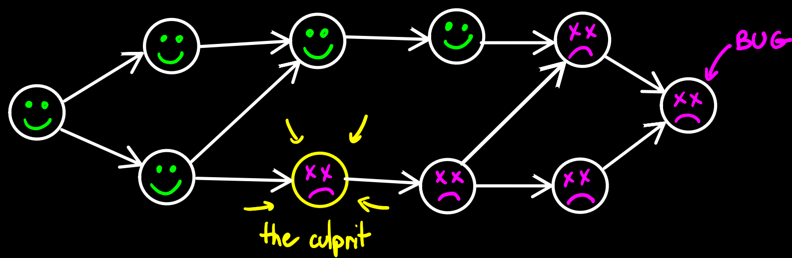
This forms a Directed Acyclic Graph (DAG), where the vertices are the project states, also named commits.

PART I
GIT BISECT

PROBLEM: FINDING A REGRESSION



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Question Which commit has originally introduced the bug?

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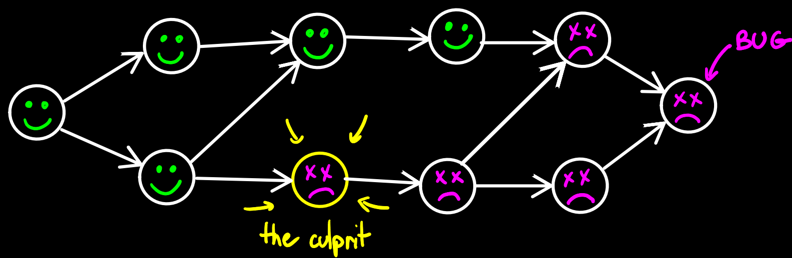

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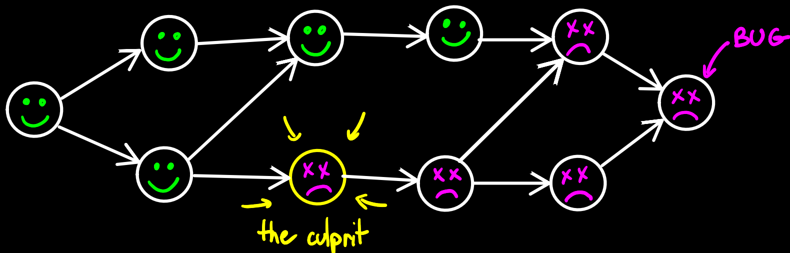

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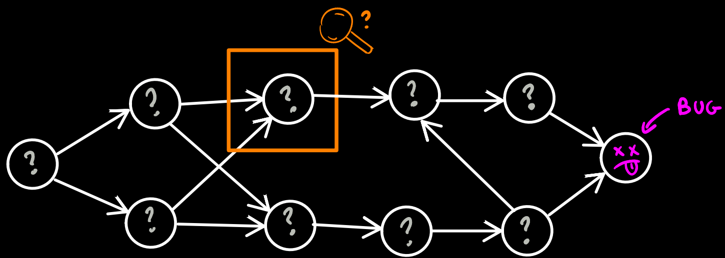
Question Which commit has originally introduced the bug?

Assumptions
- If a parent of a commit is bugged, then the commit is bugged.
- Only one commit has introduced the bug, namely the faulty commit (or regression)

HOW TO INVESTIGATE

Unique operation: QUERY of a commit with unknown status



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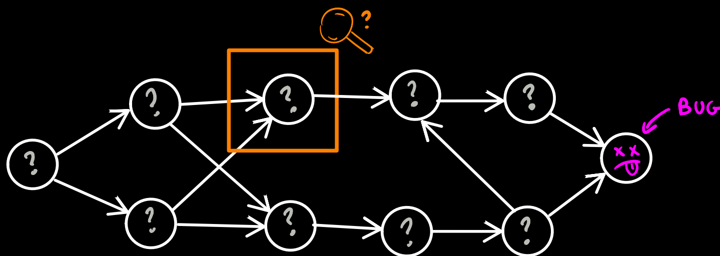
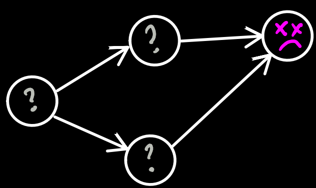


If bugged,

then the faulty commit is an ancestor of this commit



ancestor of a vertex v =
v or
an ancestor of a parent of v

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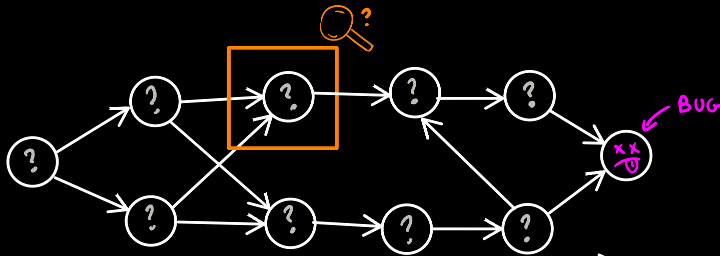

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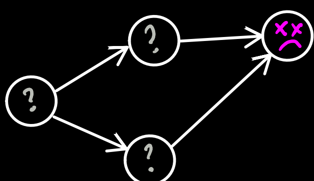

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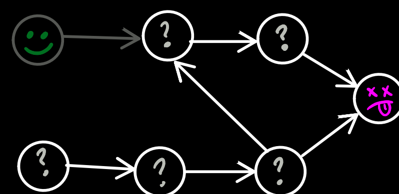

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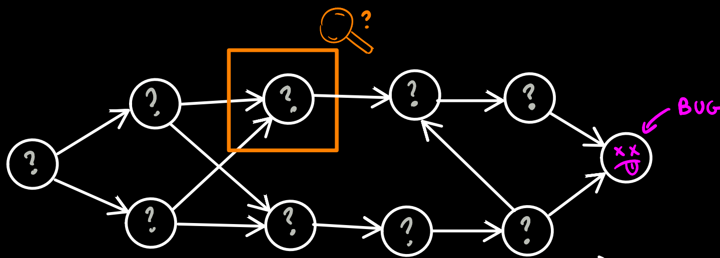

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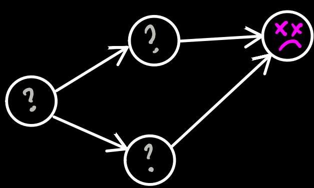

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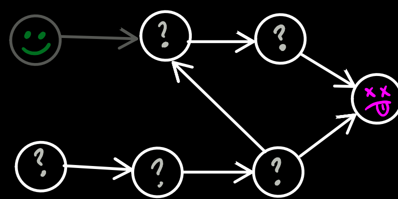
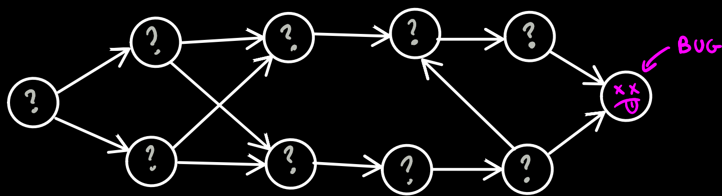


The faulty commit is found whenever there remains only 1 suspect.

REGRESSION SEARCH PROBLEM

Input: a DAG where each vertex has an unknown status, except one, which is bugged.



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(the faulty commit can be any ancestor of the bugged vertex)

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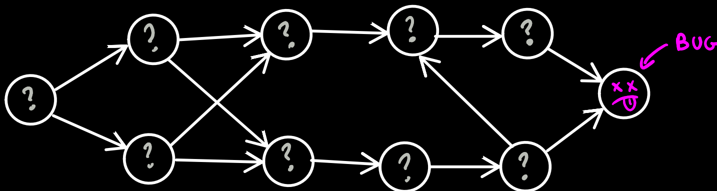

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In real life, queries are costly.

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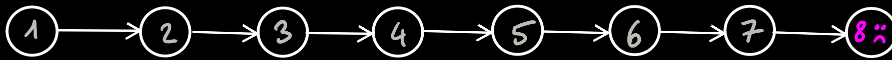

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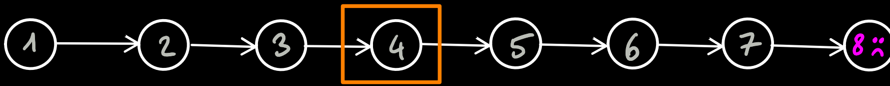

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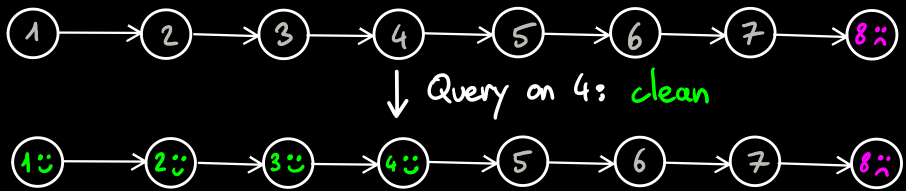

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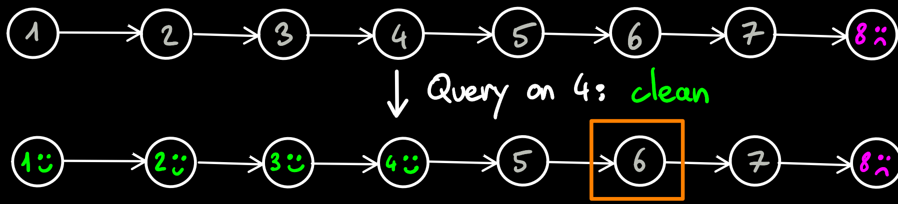

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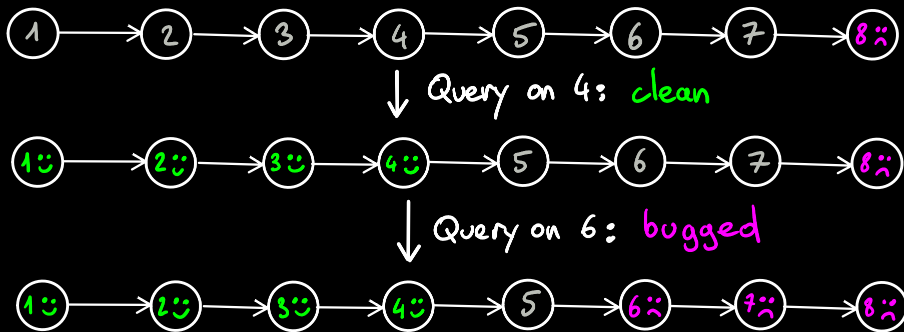

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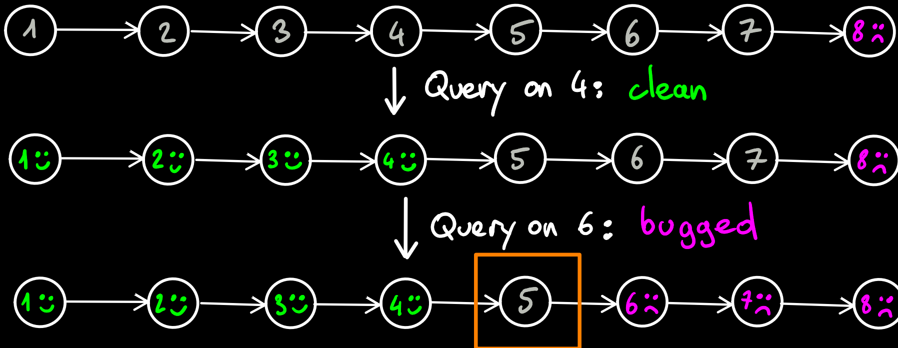

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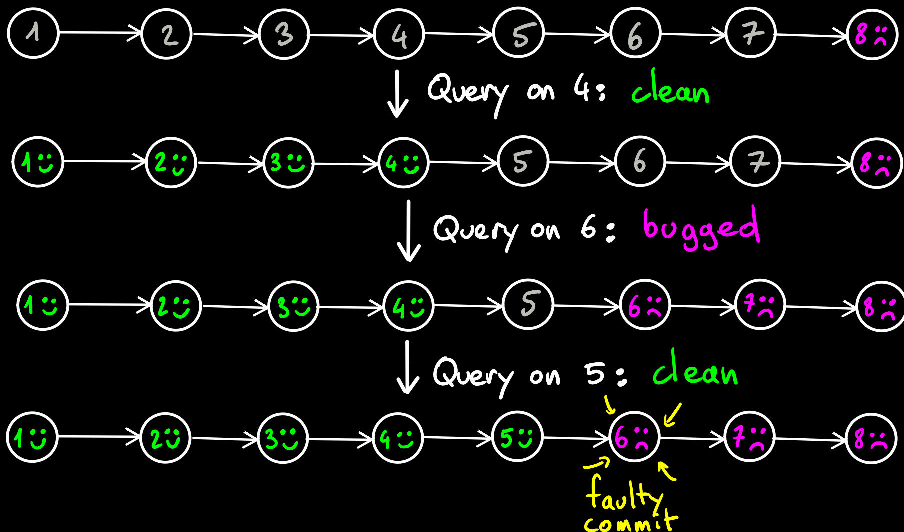

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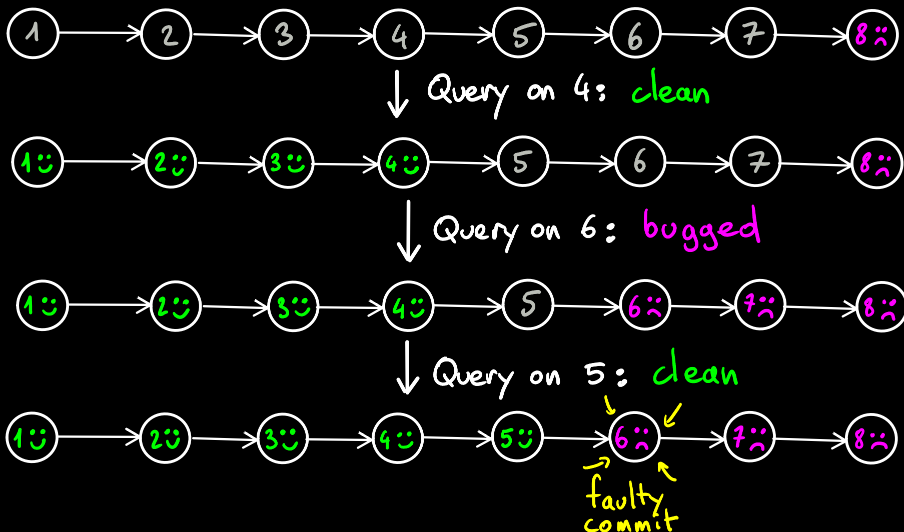

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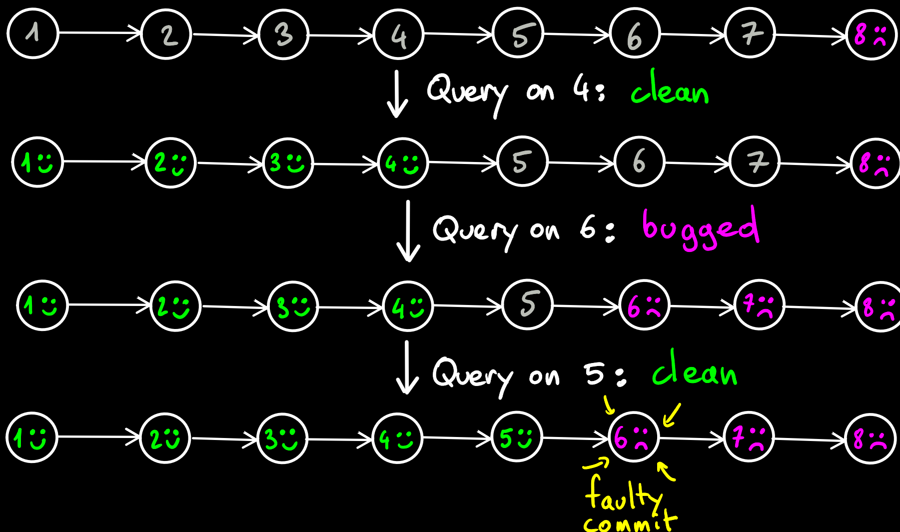

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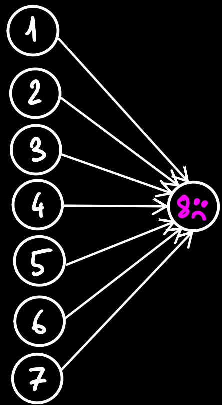
More generally, number of queries in an optimal strategy in a chain of length n = $\lceil \log_2(n) \rceil$

SECOND EXAMPLE : OCTOPUSES



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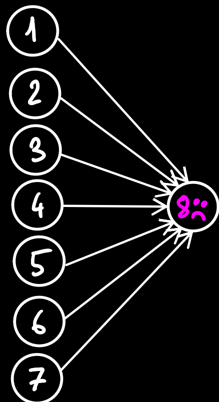

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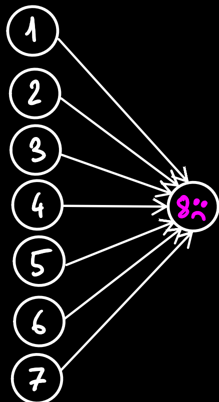

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More generally, number of queries in an optimal strategy in an octopus of size n = $n-1$

THE GIT BISECT ALGORITHM

git uses a heuristic to find the faulty commit: git bisect

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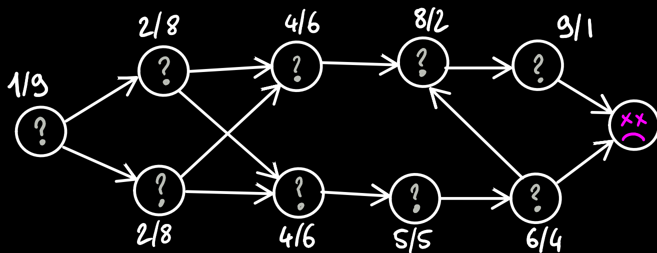

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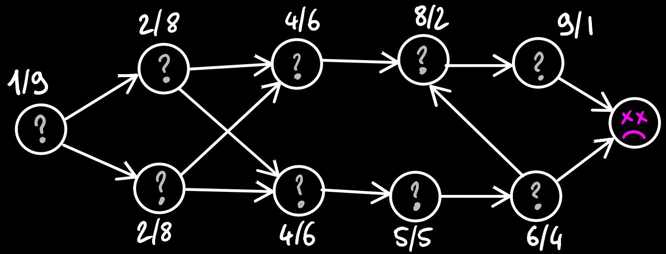

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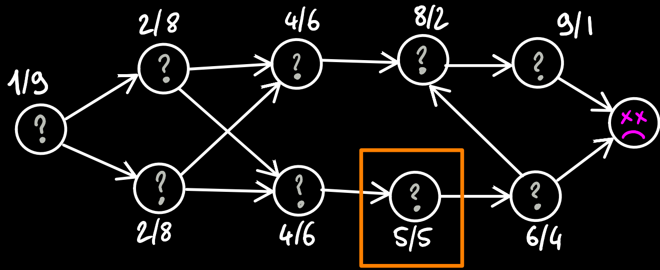

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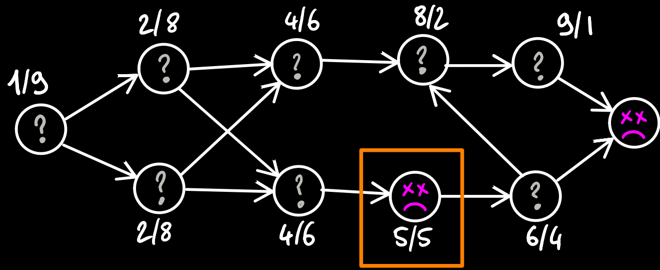

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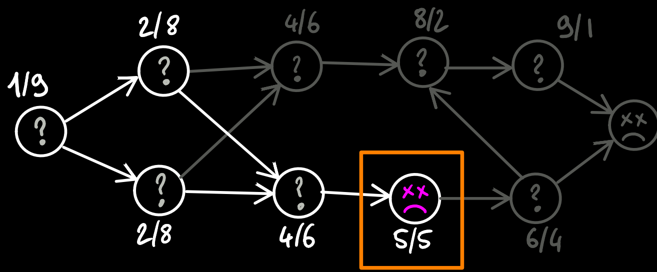

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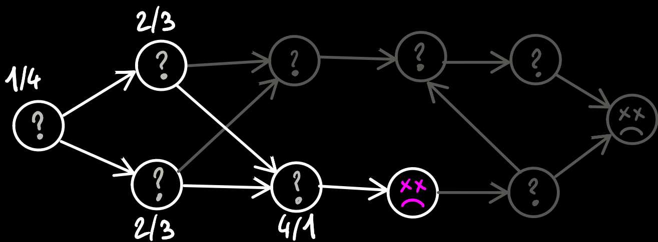

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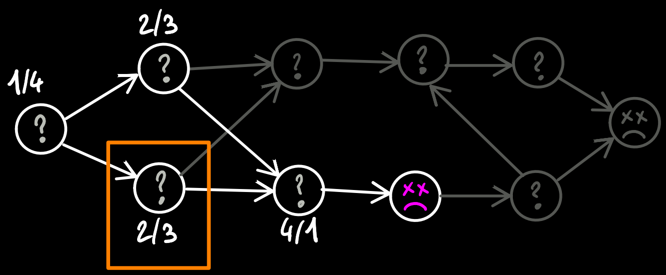

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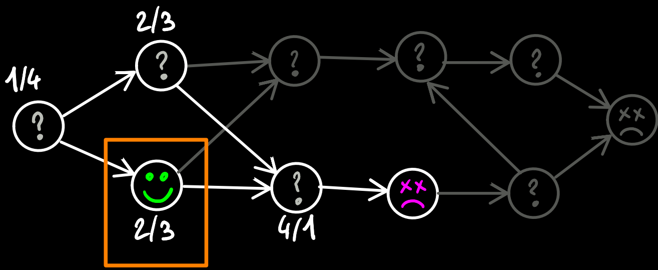

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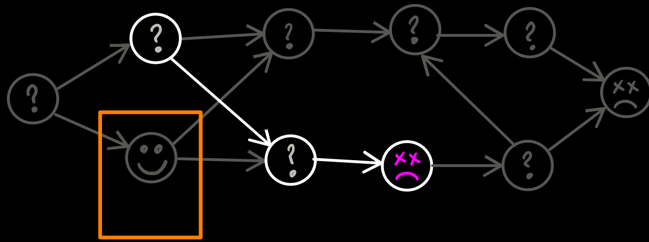

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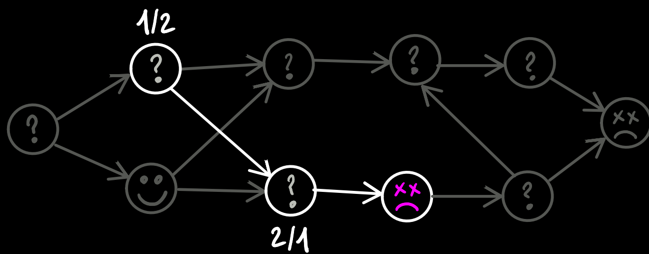

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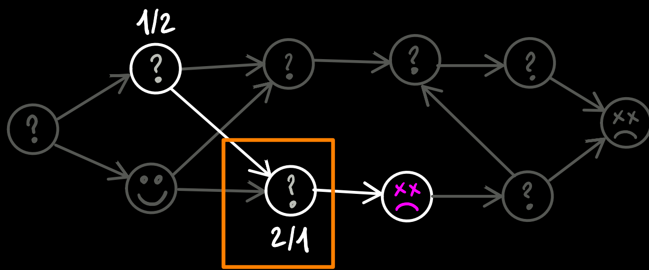

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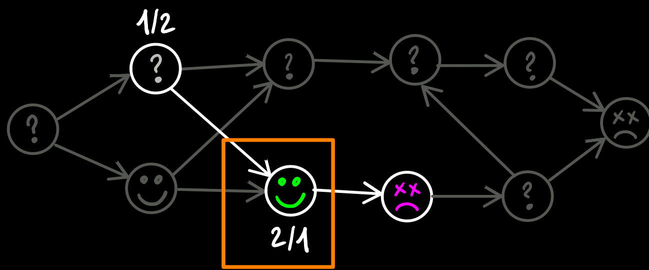

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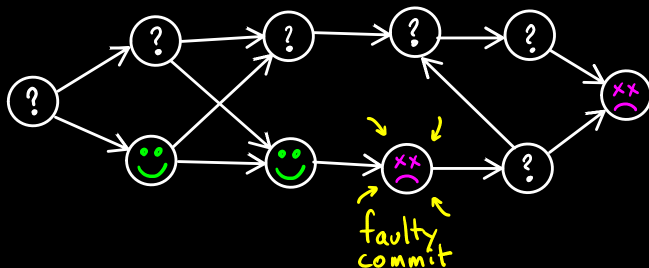

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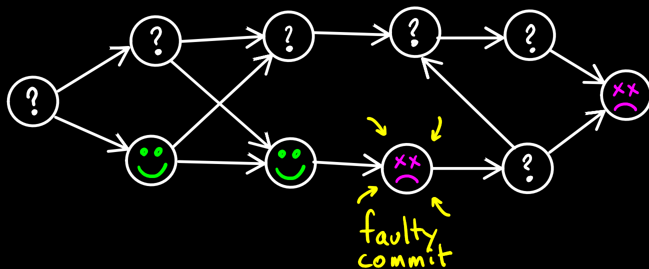

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[Carmo Donadelli Kohayakawa Laber 2004]

[We've also proved it!]

But is it really that bad?

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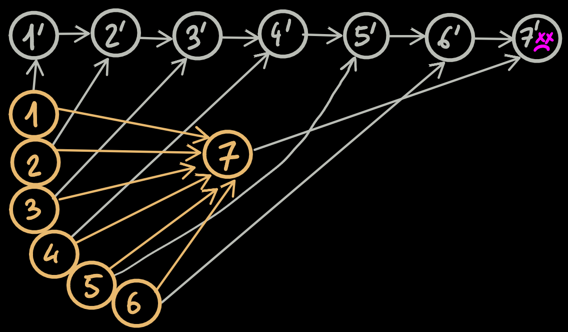

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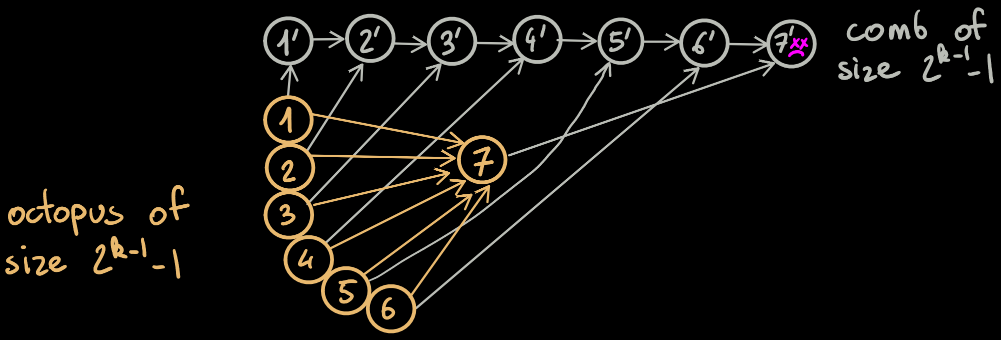

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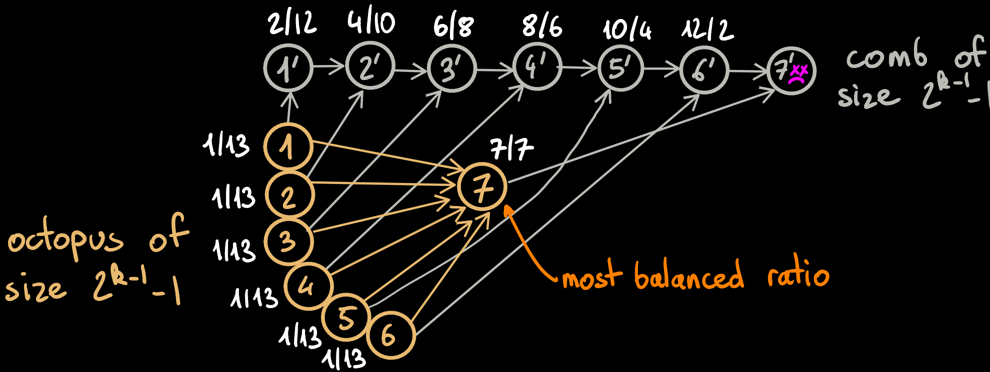
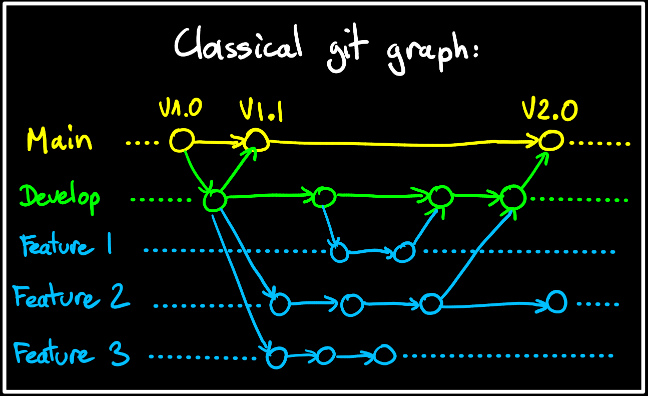


BACK TO REALITY?

Octopus substructures are unrealistic



Usually, we never merge more than 2 branches.

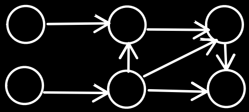
(Otherwise it is called an octopus merge )

BINARY DAGS

Definition

binary DAG = DAG where the vertices have indegree ≤ 2

Ex:



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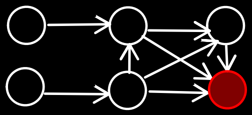

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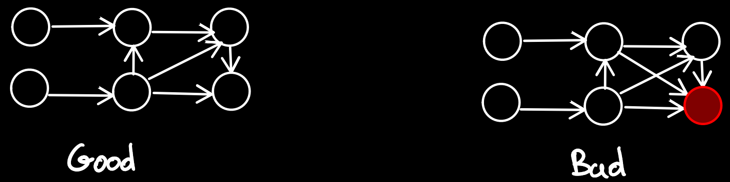
Ex:



Theorem git bisect is a $\dfrac{1}{\log_2\left(\frac{3}{2}\right)}$-approximation algorithm when it is used on binary DAGs.

$$\frac{1}{\log_2\left(\frac{3}{2}\right)} \simeq 1{,}71$$ is the optimal constant.

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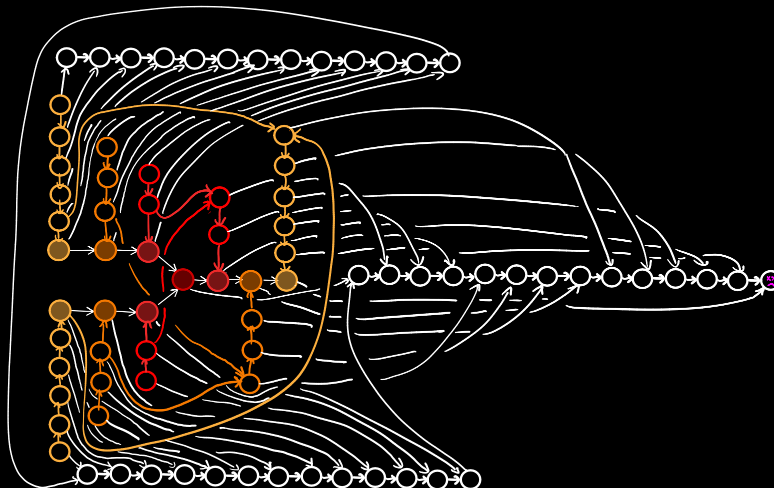
Example of a mean 😈 binary DAG



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→ new algorithm : golden bisect

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$\longrightarrow$ new algorithm : ☆ golden bisect ☆

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Theorem

golden bisect is a $\frac{1}{\log_2(\phi)}$ - approximation algorithm for binary DAGs, where ϕ = golden ratio.
$= (1+\sqrt{5})/2$

$\frac{1}{\log_2(\phi)} \simeq 1{,}44$ is the optimal constant.

QUESTION

Have we really proved that git bisect is bad?

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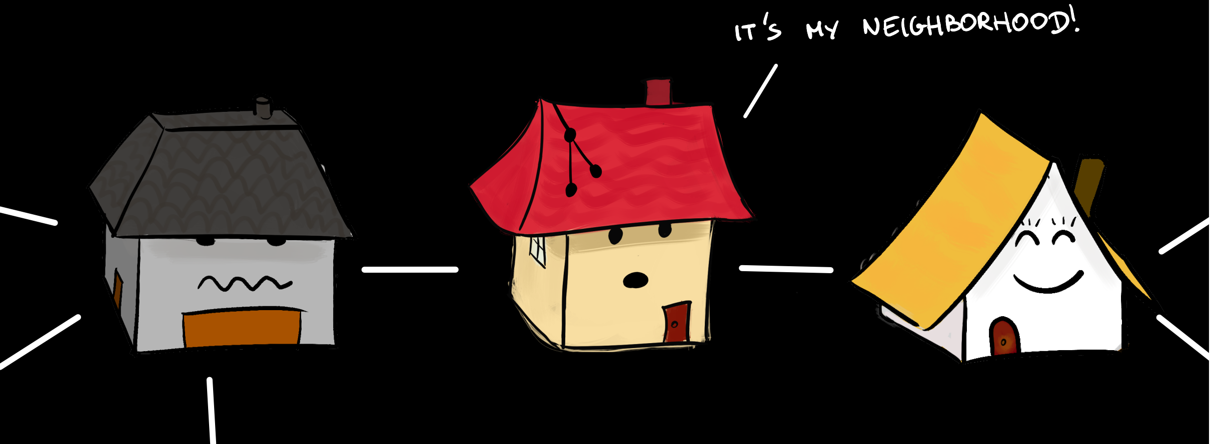
More relevant(?) question	What is the average-case complexity of git bisect?

This question calls for many more, notably <u>what is a random Git graph?</u>

Part II
Random Git Graphs

ongoing work



WHICH GRAPHS TO CONSIDER ?

In git, every DAG
without restriction
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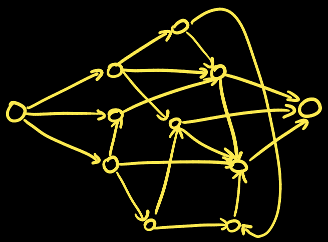

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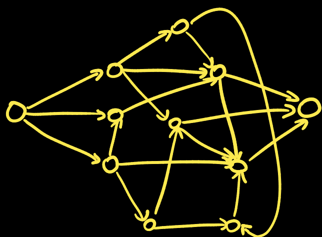

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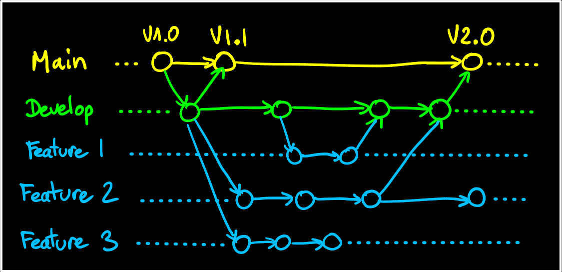

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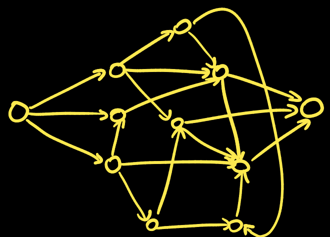

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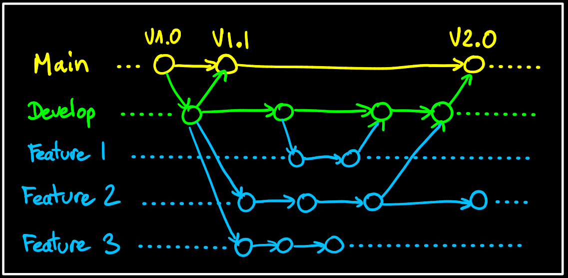


In the following, we consider a simple workflow but widely used in industry: the <u>feature branch workflow</u>

GIT GRAPH

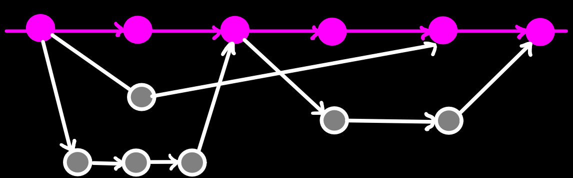
DEFINITION

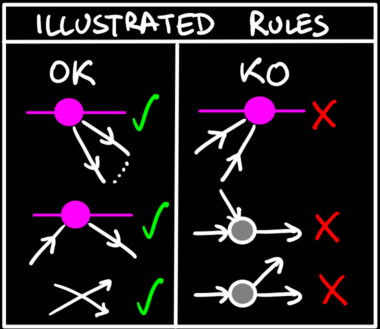
(feature branch)
Git graph = DAG with

- a main branch (path of magenta vertices)
- 0, 1 or several feature branches, paths of ≥ 1 white vertices starting and ending on magenta vertices
- indegree ≤ 2 for all vertices

previously defined in [Lecoq 2024]

e.g.





OBJECTIVES

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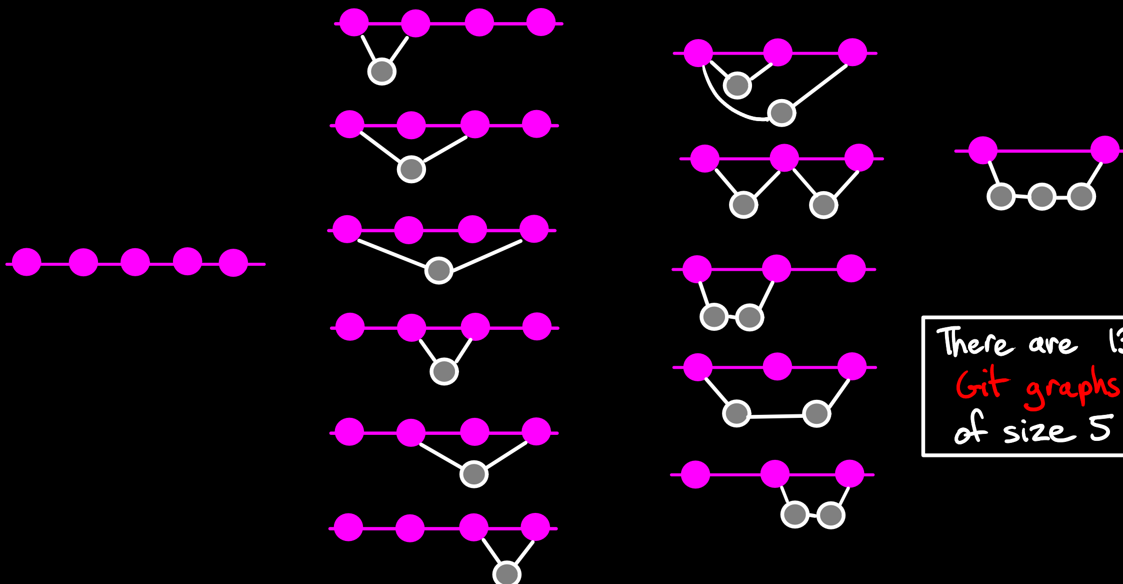
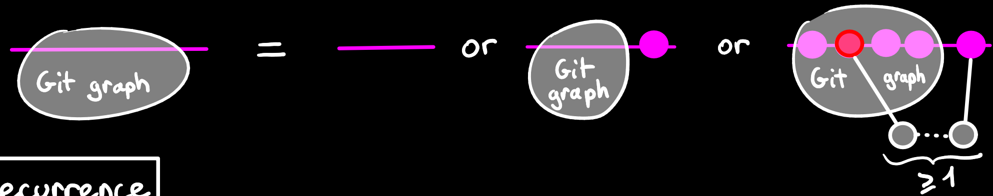


There are 13 Git graphs of size 5

RECURSIVE DECOMPOSITION

Decomposition



Recurrence

$$g_{n,k} = g_{n-1,k-1} + \sum_{\ell \geq 1} (k-1)\, g_{n-1-\ell,\, k-1} \quad \text{for } n \geq 1$$

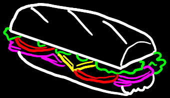
where $g_{n,k} :=$ number of Git graphs with n vertices, k of them being magenta

Differential Equation for the Generating Function

$$G(z,u) = 1 + zu\,G(z,u) + \frac{z^2u^2}{1-z}\frac{\partial G}{\partial u}(z,u)$$

where $G(z,u) = \sum_{n \geq 0} \sum_{k \geq 0} g_{n,k}\, z^n u^k$

⚠ $G(z,u)$ is not analytic.

 SANDWICHING $g_{n,k}$

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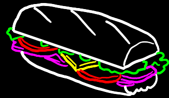
$$\boxed{\binom{n-k-1}{k-2}(k-1)! \leq g_{n,k} \leq \binom{n-2}{k-2}(k-1)! \quad \text{if } k \leq \frac{n+1}{2}}$$



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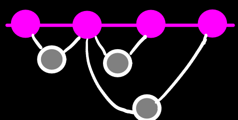

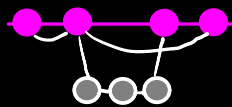

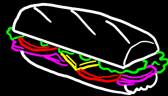

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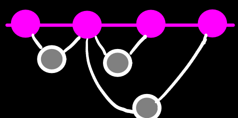
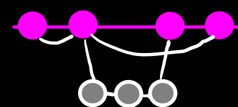
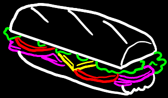




$$\boxed{\frac{(k-1)!}{(2k-n-1)!} \leq g_{n,k} \leq \binom{n-2}{k-2}\frac{(k-1)!}{(2k-n-1)!} \quad \text{if } k > \frac{n+1}{2}}$$



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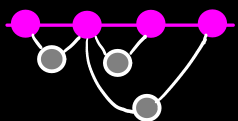

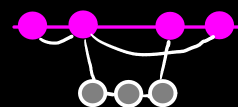

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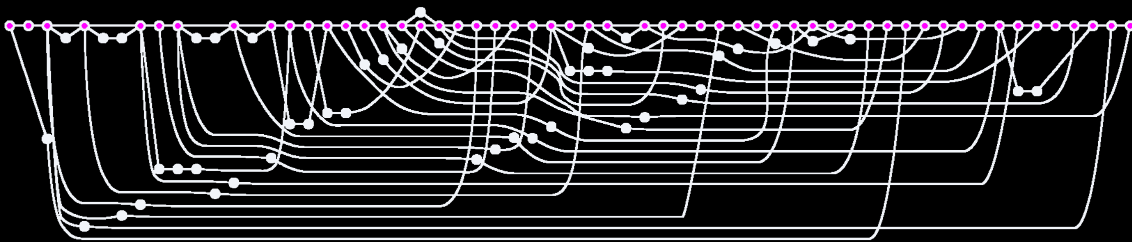
<u>Consequence:</u> $\sum_{n,k} g_{n,k} z^n u^k$ is not analytic, but some asymptotic analysis can be done.

MOST GIT GRAPHS LOOK ALIKE

Theorem

In a Git graph of size n taken uniformly at random, the number of magenta vertices is $\frac{n}{2} + o(n)$



random Git graph of size 100

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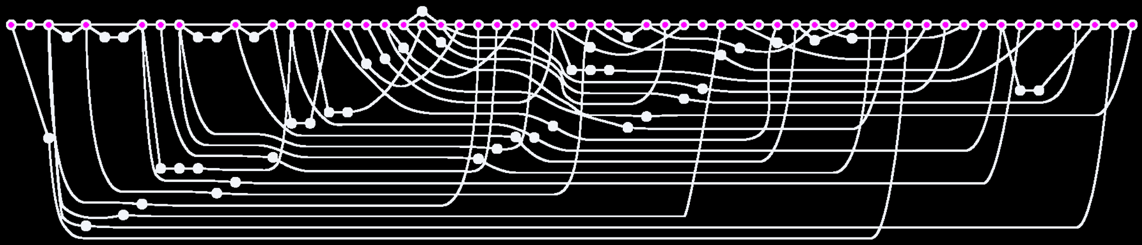

random Git graph of size 100

Sampling a Git graph is more interesting if we fix size n and number of magenta vertices

TRANSFORMING THE EQUATION

Recurrence	$g_{n,k} = g_{n-1,k-1} + \sum_{\ell \geq 1} (k-1)\, g_{n-1-\ell,\, k-1}$
Differential Equation	$G(z,u) = 1 + z u\, G(z,u) + \frac{z^2 u^2}{1-z} \frac{\partial G}{\partial u}(z,u)$

Usual trick:

Ordinary Generating Function $\underbrace{\sum_{n,k \geq 0} g_{n,k} z^n u^k}_{G(z,u),}$

not analytic ✗

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Borel transform on u

$$\tilde{G}(z,u) = \sum_{n,k \geq 0} \frac{g_{n,k}}{k!} z^n u^k$$

analytic ✓

and

Differential Equation for $\tilde{G}$
$\dfrac{\partial \tilde{G}}{\partial u} = z \tilde{G} + \dfrac{z^2 u}{1-z} \dfrac{\partial \tilde{G}}{\partial u}$

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this can be solved!

Theorem

$$\tilde{G}(z,u) = \left(1 - \frac{z^2u}{1-z}\right)^{-\frac{1-z}{z}}$$

How can it be exploited?

Different Perspective

Is there a combinatorial explanation for the formula

$$\tilde{G}(z,u) = \left(1 - \frac{z^2 u}{1-z}\right)^{-\frac{1-z}{z}} \quad ?$$

DIFFERENT PERSPECTIVE

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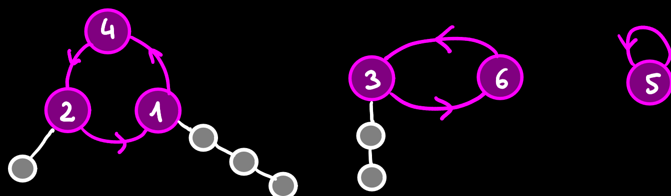
Different Perspective

Definition

cyclarium = set of cycles of magenta vertices labeled from 1 to k where a chain of white unlabeled vertices is attached to each magenta vertex, except to the ones having the largest label in their cycles.

e.g.:



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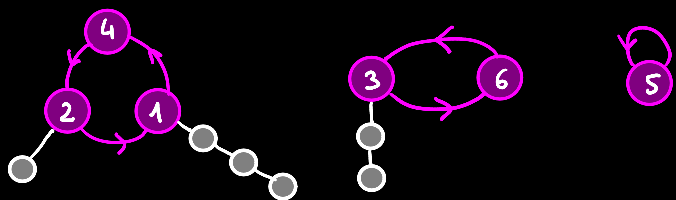

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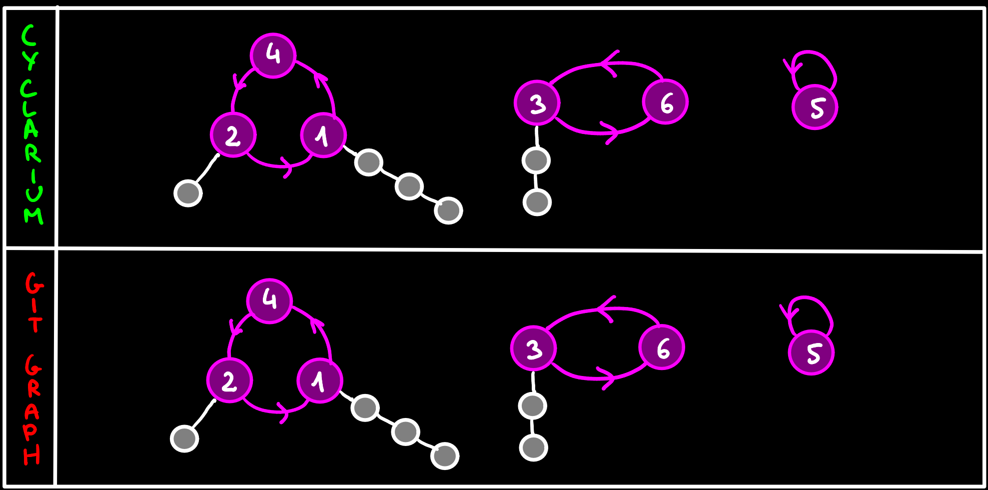
It's the generating function of cyclariums!

BIJECTION

Proposition

There is a bijection from cycłariums to Git graphs:



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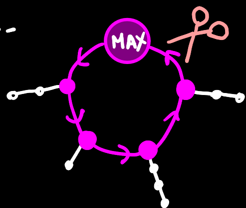

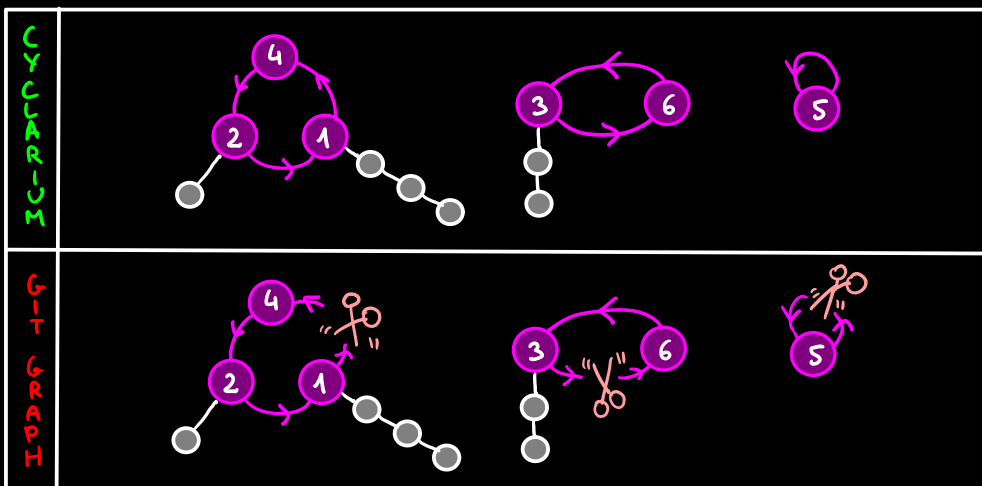

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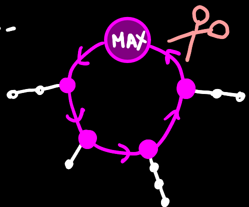

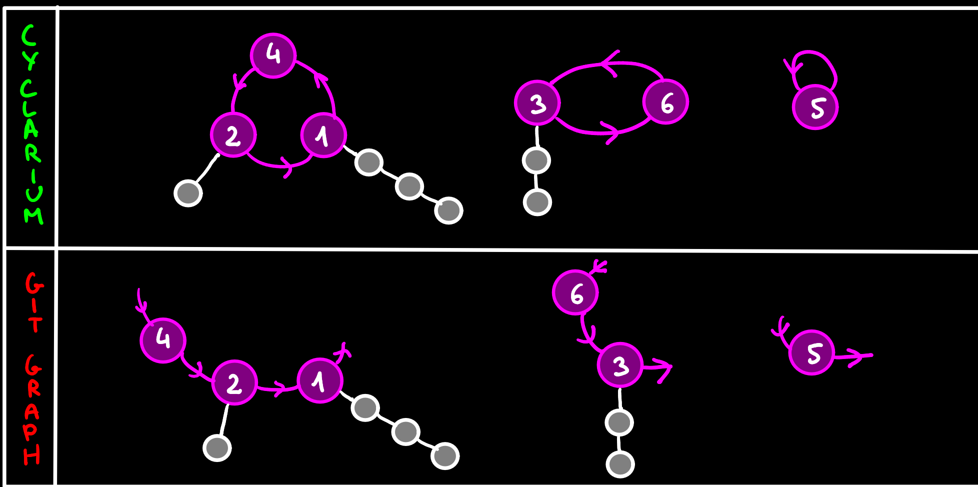

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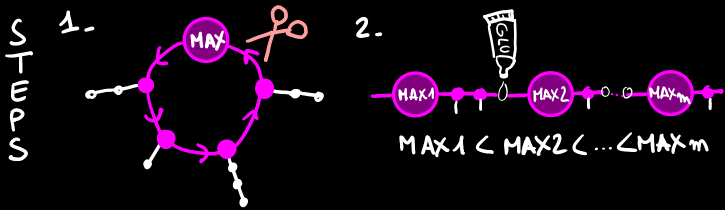

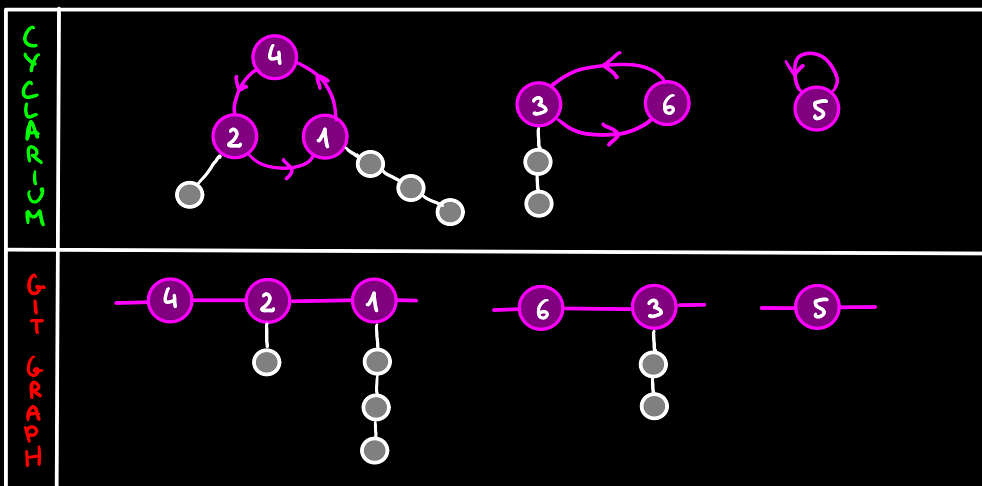

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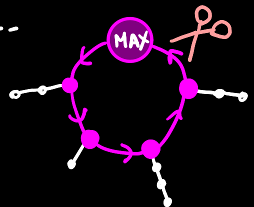

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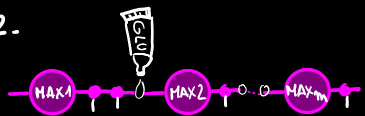
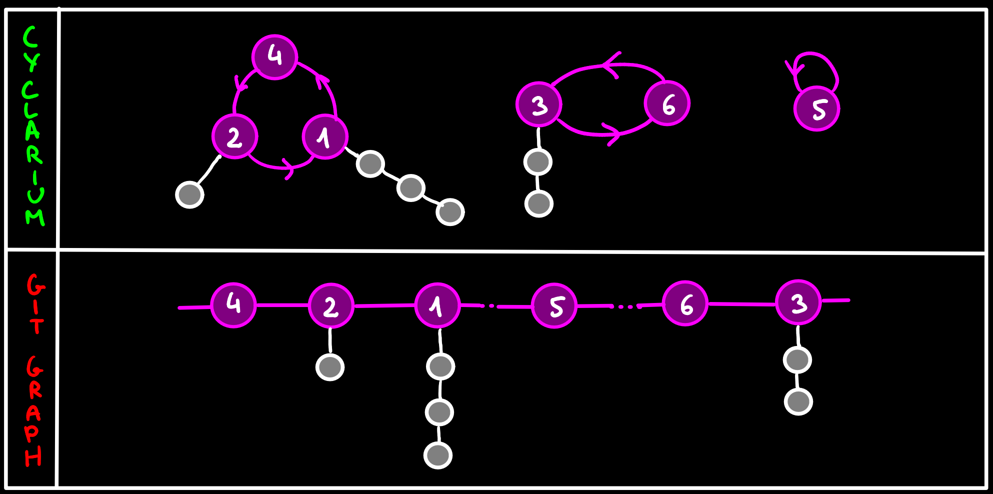


$MAX1 < MAX2 < \dots < MAXm$



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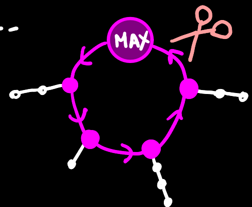

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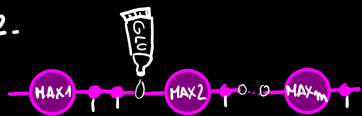
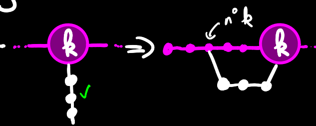


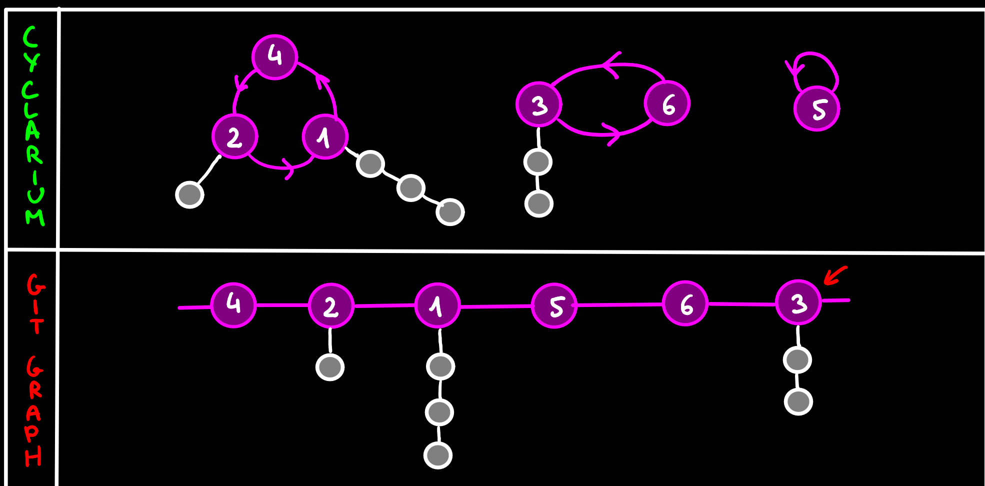
MAX1 < MAX2 < ... < MAXm

3. Right to left :

3a.



3b. Label removing + shift



BIJECTION

Proposition

There is a bijection from cyclariums to Git graphs:

STEPS

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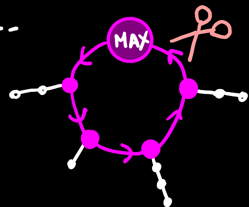

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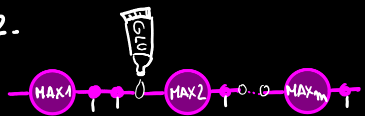

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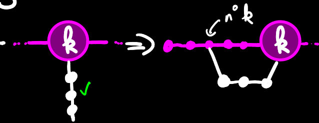

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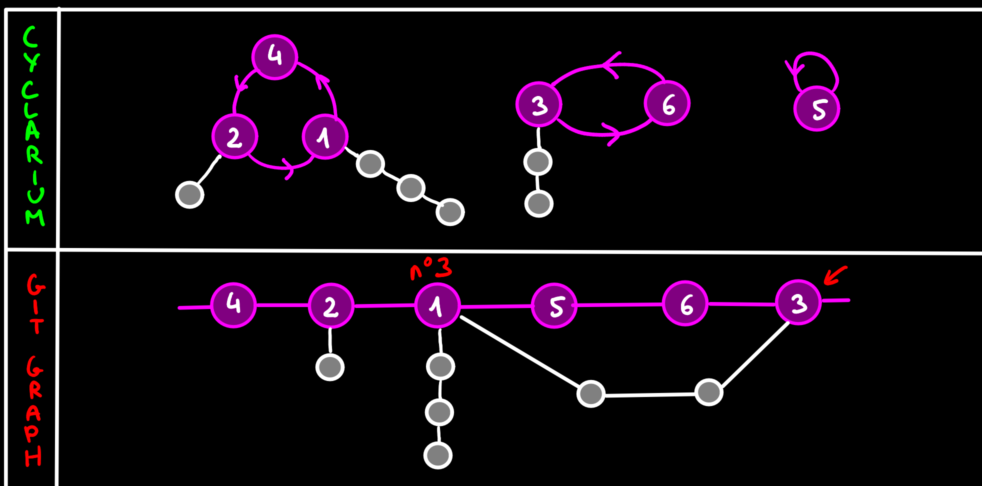

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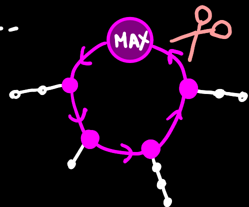

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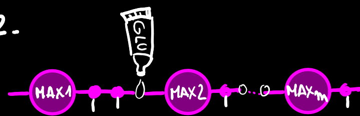

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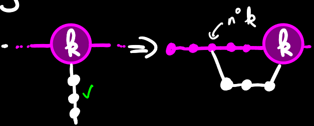

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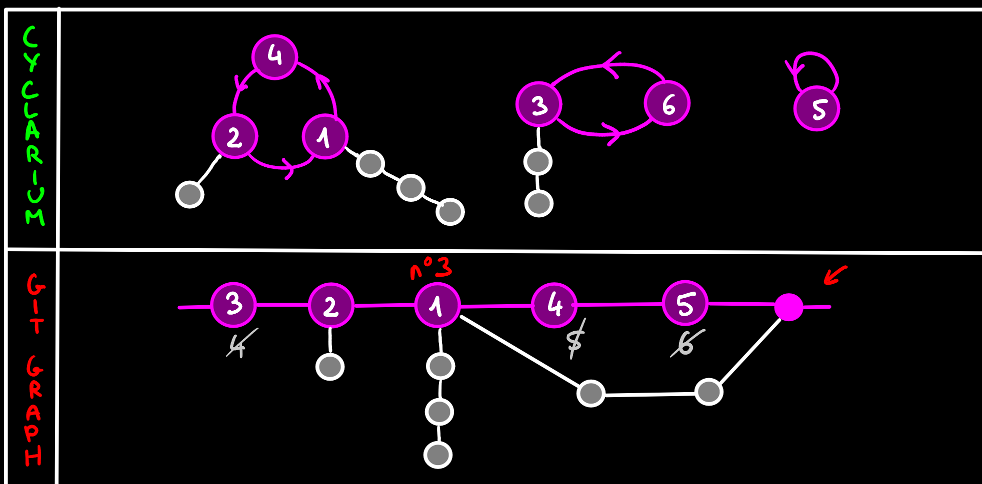

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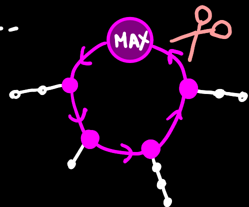

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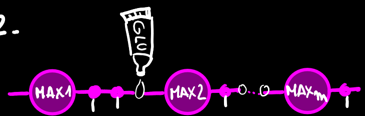

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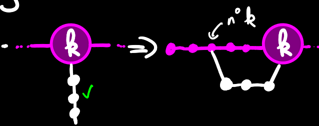

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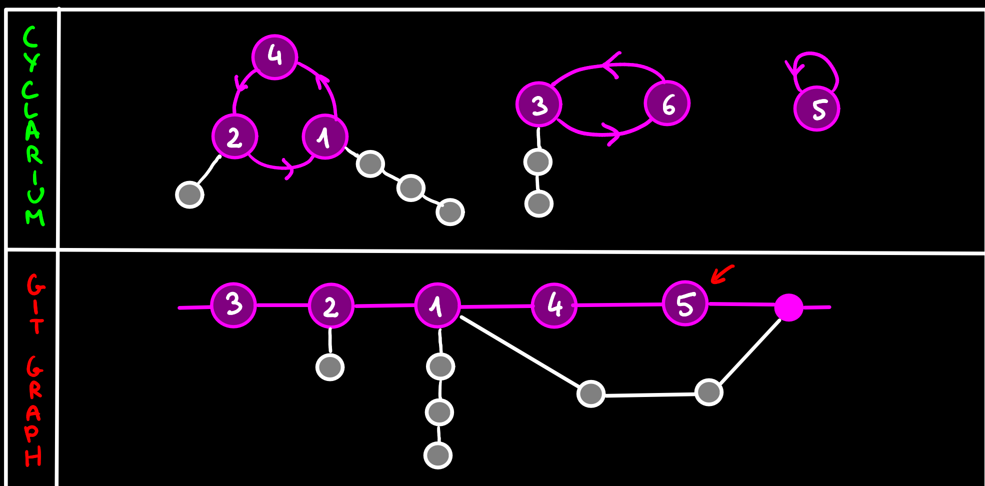

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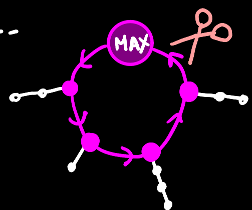

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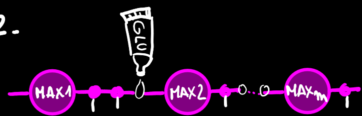

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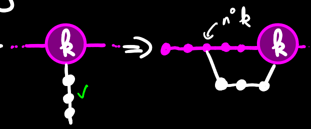

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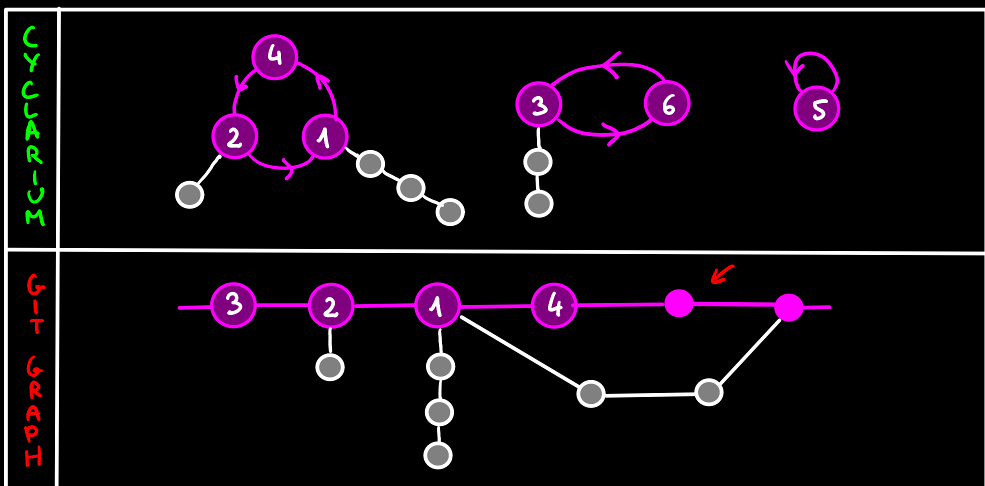

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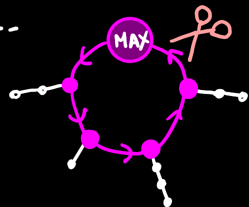

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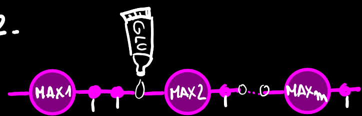

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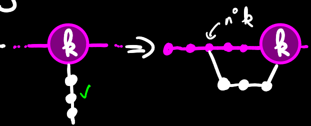

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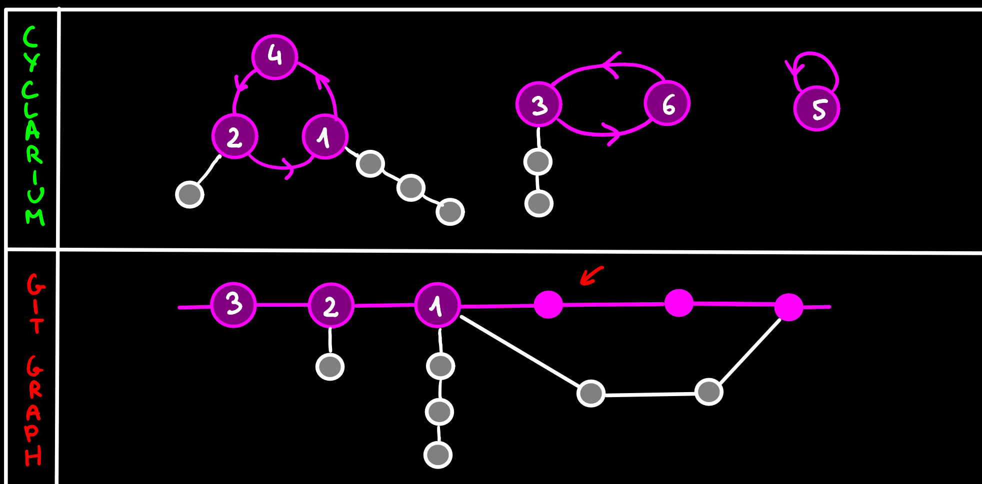

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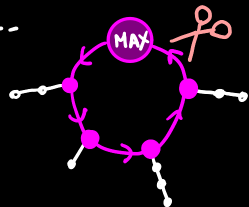

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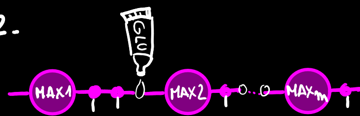

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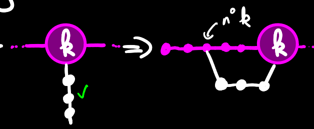

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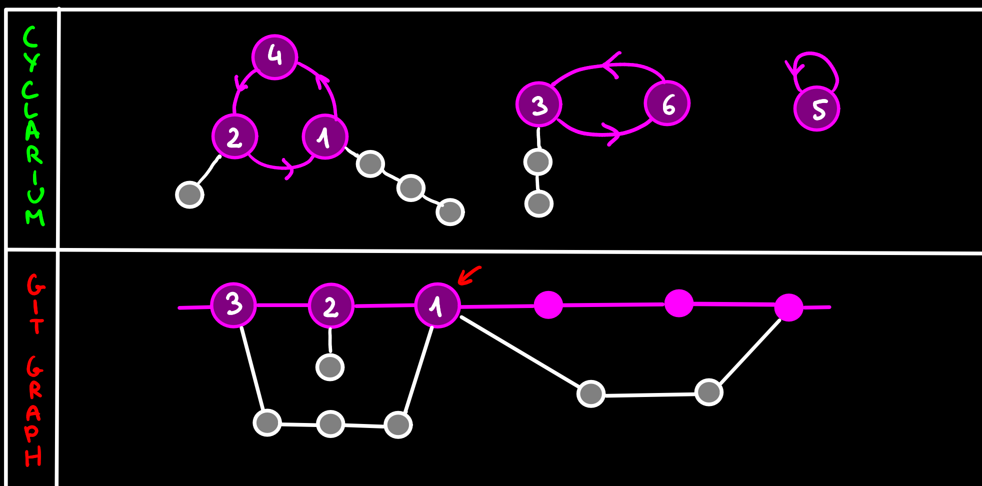

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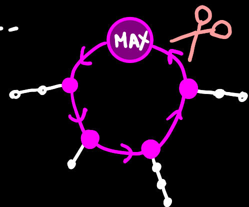

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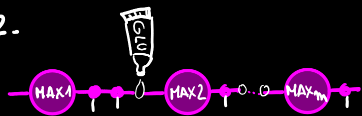

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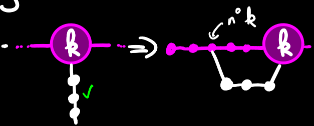

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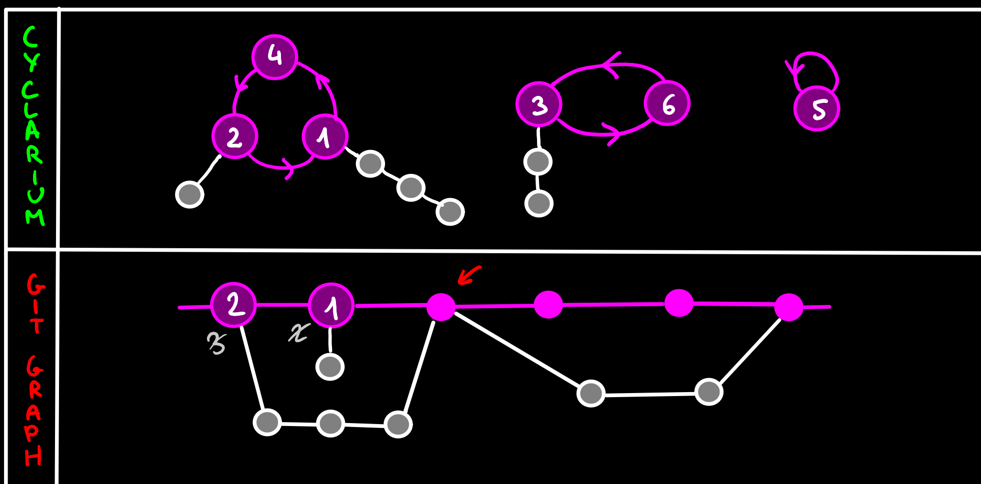

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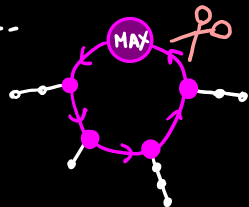

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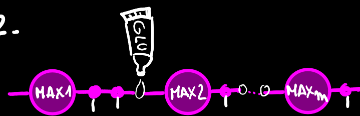

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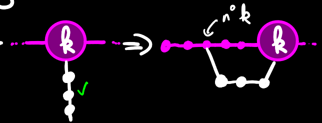

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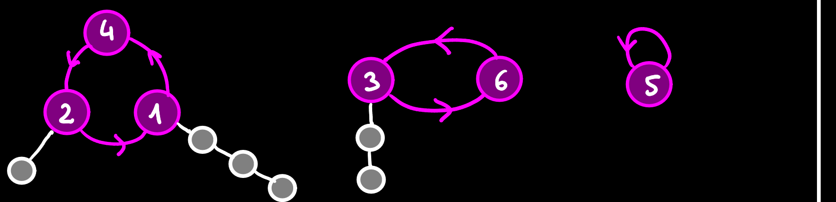
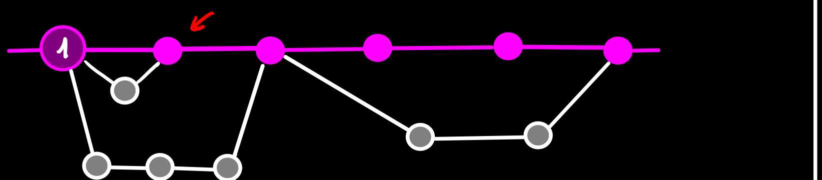
CYCLARIUM



GIT GRAPH



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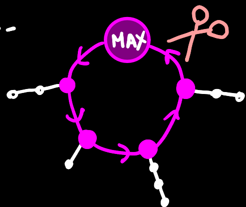

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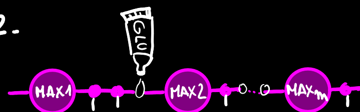

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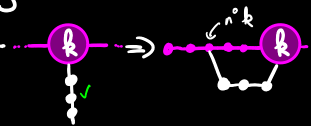

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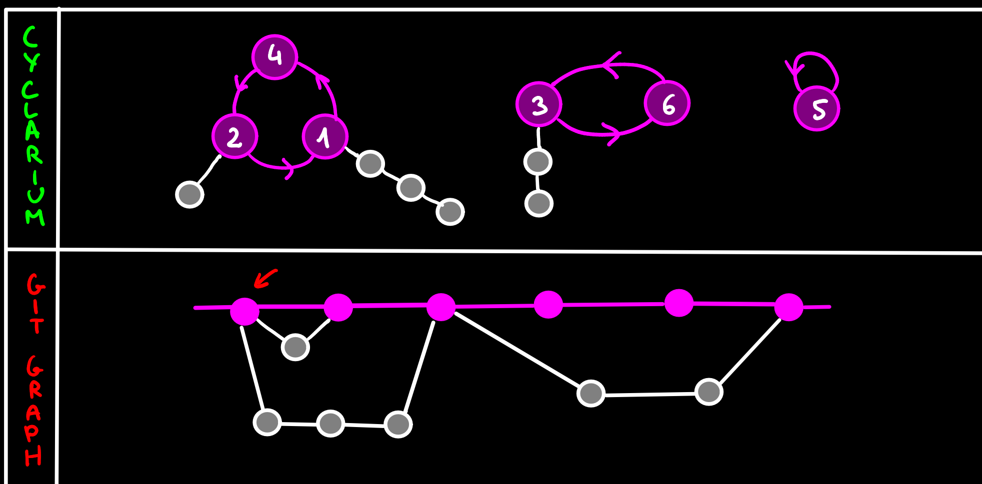

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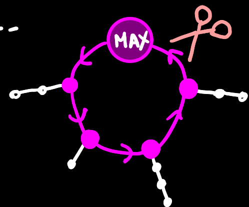

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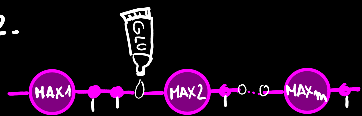

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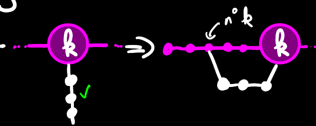

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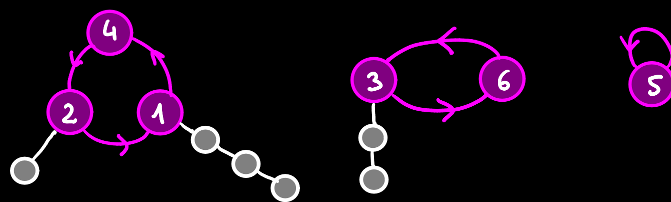
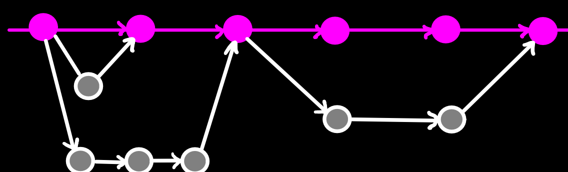


GIT GRAPH



BIJECTION

Proposition

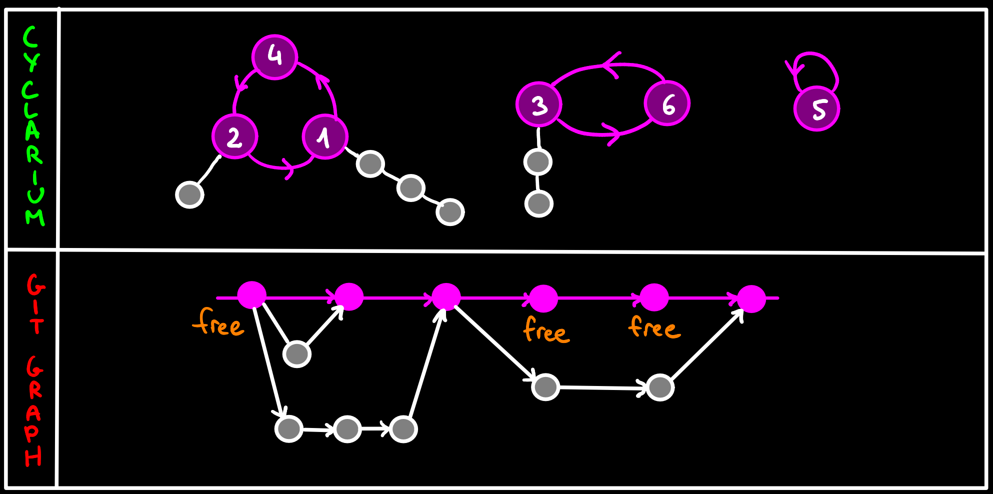
There is a bijection from cyclariums to Git graphs:
sending

vertices $\longrightarrow$ vertices

magenta vertices $\longrightarrow$ magenta vertices

cycles $\longrightarrow$ free vertices
i.e magenta vertices of indegree ≤ 1

cycle lengths $\longrightarrow$ gaps between free vertices



A NICE FORMULA

Proposition

There is a bijection from cyclariums to Git graphs: sending

vertices ⟶ vertices

magenta vertices ⟶ magenta vertices

cycles ⟶ free vertices
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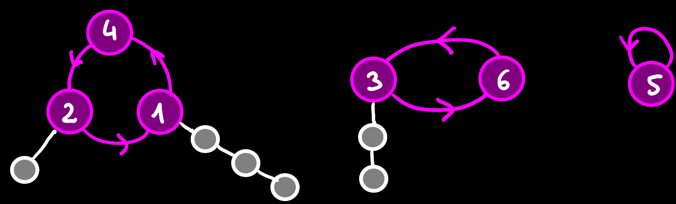
CYCLARIUM



Corollary

$$g_{n,k} = \sum_{f=1}^{k-1} \begin{bmatrix} k \\ f \end{bmatrix} \binom{n-k-1}{k-f-1} \qquad (k < n)$$

where $g_{n,k}$ = number of Git graphs counted by vertices & magenta vertices

and $\begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$ = (unsigned) Stirling number of 1st kind

RANDOM MODEL

"Boltzmann model" (exponential in u, ordinary in z)

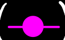
Fix $z > 0^*$ and $u > 0^*$.

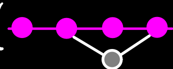
We wish to draw a Git graph γ with a weight proportional to $z^{\#\text{vertices in } \gamma} \dfrac{u^{\#\text{magenta vertices in } \gamma}}{(\#\text{magenta vertices in } \gamma)!}$

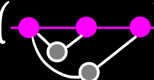
(Size is not fixed)

Examples

$\mathbb{P}($  $) \propto 1$

$\mathbb{P}($  $) \propto z u$

$\mathbb{P}($  $) \propto z^5 \dfrac{u^4}{24}$

$\mathbb{P}($  $) \propto z^5 \dfrac{u^3}{6}$

$*$: in the disk of convergence of $\tilde{G}$

RANDOM MODEL

"Boltzmann model" (exponential in u, ordinary in z)

Fix $z > 0$* and $u > 0$*.

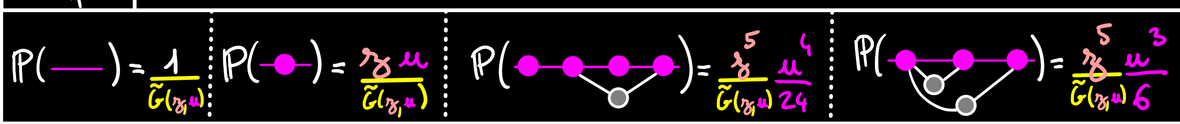
We wish to draw a Git graph γ with a weight ~~proportional~~ equal to

$$\frac{z^{\#\text{vertices in } \gamma}}{\tilde{G}(z,u)} \; \frac{u^{\#\text{magenta vertices in } \gamma}}{(\#\text{magenta vertices in } \gamma)!}$$

(Size is not fixed)

where $$\tilde{G}(z,u) = \sum_{n,k \geq 0} \frac{g_{n,k}}{k!} z^n u^k = \left(1 - \frac{z^2 u}{1-z}\right)^{-\frac{1-z}{z}}.$$

Examples



*: in the disk of convergence of $\tilde{G}$

RANDOM MODEL

Proposition

Let γ be a random Git Graph sampled with respect to the previous Boltzmann model, conditioned to have size n

$$\mathbb{E}(\#\text{magenta vertices}(\gamma)) \sim \frac{1-\rho_u}{2-\rho_u}\, n$$

$$\mathbb{V}(\#\text{magenta vertices}(\gamma)) \sim \frac{\rho_u(1-\rho_u)}{(2-\rho_u)^3}\, n$$

where $\rho_u = \dfrac{\sqrt{1+4u}-1}{2u}$

<u>Proof</u> : Transfer Theorem from $\tilde{G}(z,u) = \left(1 - \dfrac{z^2 u}{1-z}\right)^{-\frac{1-z}{z}}$

RANDOM MODEL

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where $\rho_u = \dfrac{\sqrt{1+4u}-1}{2u}$

<u>Proof</u>: Transfer Theorem from $\tilde{G}(z,u) = \left(1 - \dfrac{z^2 u}{1-z}\right)^{-\frac{1-z}{z}}$

<u>Consequence</u>: A random generator for Git graphs with $\approx n$ vertices and $\simeq k$ magenta vertices $\left(k \leq \frac{n}{2}\right)$

1- Tune u so that $\dfrac{1-\rho_u}{2-\rho_u} = \dfrac{k}{n}$

2- Tune z so that $z = \rho_u - \dfrac{1-\rho_u}{n}$

3- Make a Boltzmann sampler with parameters z and u for cyclariums.

4. Bijection to Git graphs

PERSPECTIVES ABOUT RANDOM GIT GRAPHS

→ Asymptotic behaviour

- Asymptotic equivalent of # Git graphs of size n ? ▷ Collaboration with Fang
- Limit Laws
- Phase transition ?

PERSPECTIVES ABOUT RANDOM GIT GRAPHS

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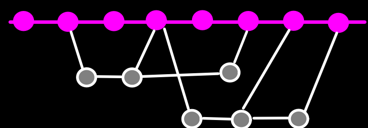
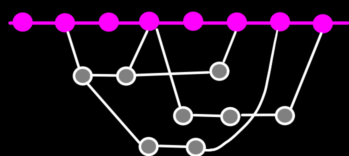
→ Other random models ▷ Collaboration with Clement + Maréchal

Phoenix graphs



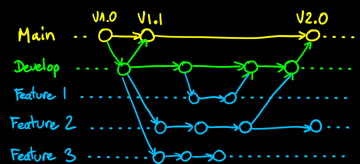
merged branches can be reborn

Fork Anywhere graphs



branches can be born anywhere but must be merged into main

More involved workflows



LINKS BETWEEN GIT GRAPHS & GIT BISECT

<u>Still to do</u>: Average-case complexity of git bisect where the input is taken w.r.t the Boltzmann distribution.

LINKS BETWEEN GIT GRAPHS & GIT BISECT

<u>Still to do</u>: Average-case complexity of git bisect where the input is taken w.r.t the Boltzmann distribution.

<u>But also</u>: - Is there a polynomial algorithm for the Regression Problem when the input is a Git graph?

(We proved that git bisect fails to be optimal for some Git graphs)

- Is the Regression Problem NP-complete when the input is binary?

- Other algorithms from Version Control Systems to be analyzed?

THANK YOU!