Julien Courtiel (PIMS, UBC)
Karen Yeats (SFU)

Terminal Chords in Connected Chord Diagrams

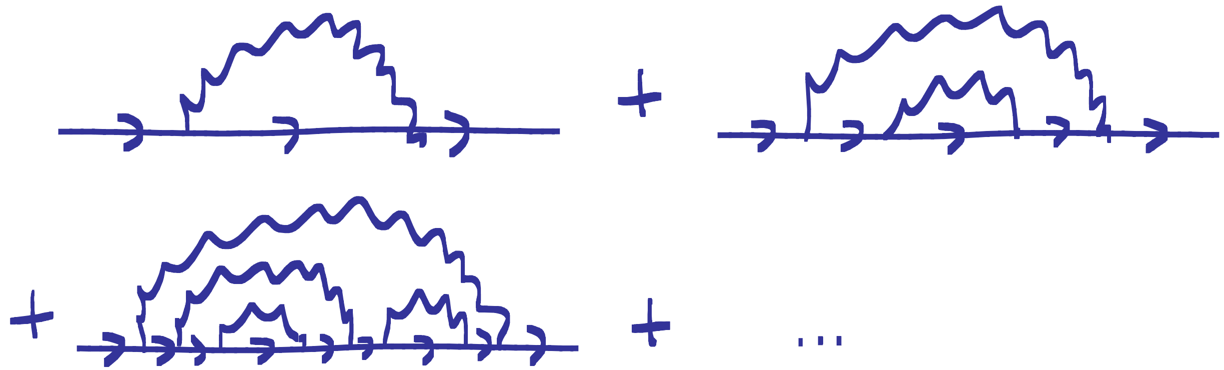
CombStruct PQFT 2016


Tom Hanks
Catherine Zeta-Jones
The Terminal
Life is waiting.

COMBINATORIAL DYSON-SCHWINGER EQUATIONS

One-loop propagator + recursive iterations

Class of Feynman graphs



Feynman rules $\downarrow$

$\downarrow$

solution of Dyson-Schwinger equations

$$G(x,L) = 1 - x\, G\left(x, \frac{\partial}{\partial(-\rho)}\right)^{-1} \left(e^{-L\rho} - 1\right) F(\rho)\Big|_{\rho=0}$$

COMBINATORIAL DYSON-SCHWINGER EQUATIONS

Theorem [Marie, Yeats]

The solution of the previous equation can be written under the form:

$$G(x,L) = 1 - \sum_{i\geq 1} \sum_{\substack{C \text{ connected chord diagram} \\ \text{with terminal chords} \\ \text{in position } t_1 < t_2 < \dots < t_k \\ \text{such that } t_1 \geq i}} \frac{L^i}{i!} x^{|C|} f_0^{|C|-k} f_{t_1-i} f_{t_2-t_1} f_{t_3-t_2} \cdots f_{t_k-t_{k-1}}$$

where

$$\frac{f_0}{\rho} + f_1 + f_2\rho + f_3\rho^2 + \dots = \text{expansion of a regularized Feynman integral}$$

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C connected chord diagram with terminal chords in position $t_1 < t_2 < \ldots < t_k$ such that $t_1 \geq i$

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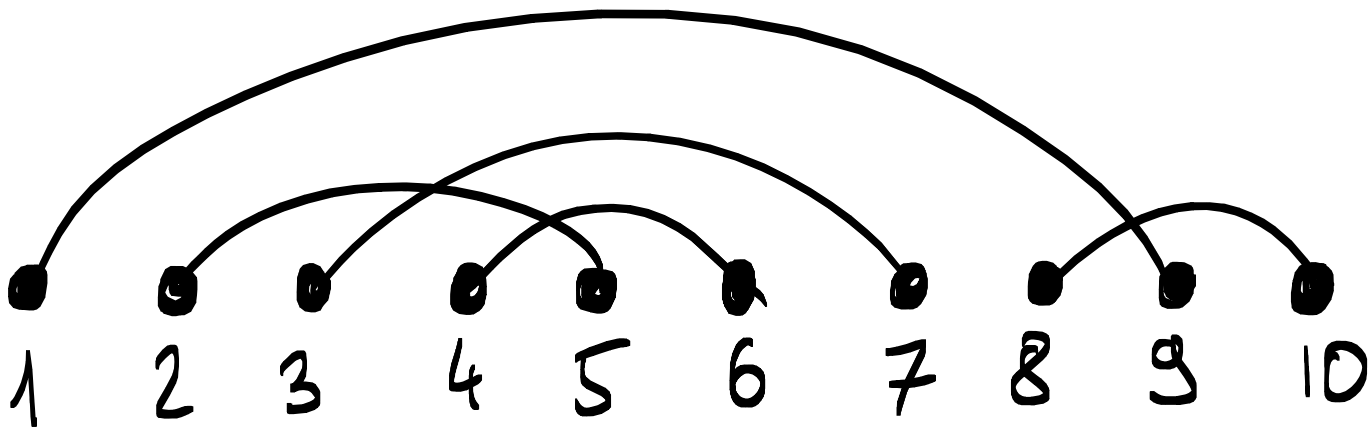
CONNECTED CHORD DIAGRAMS

Diagram of n chords = matching of $\{1, 2, \ldots, 2n\}$



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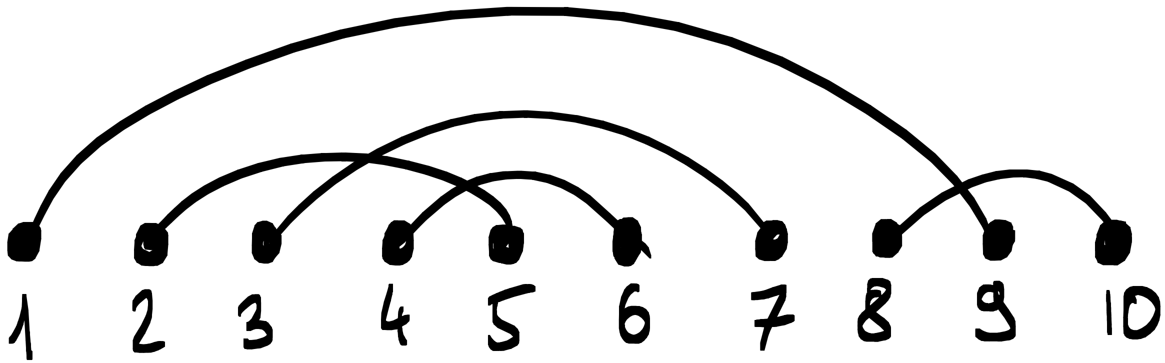

NOT CONNECTED

connected diagram : its representation is in one piece

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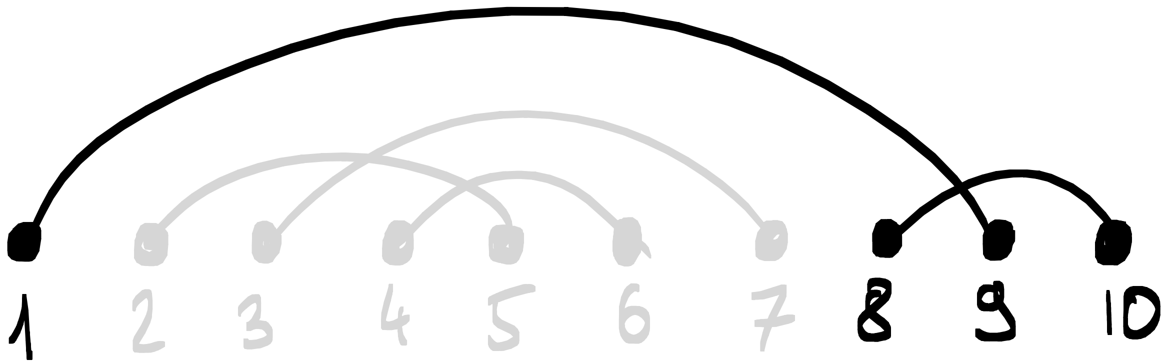

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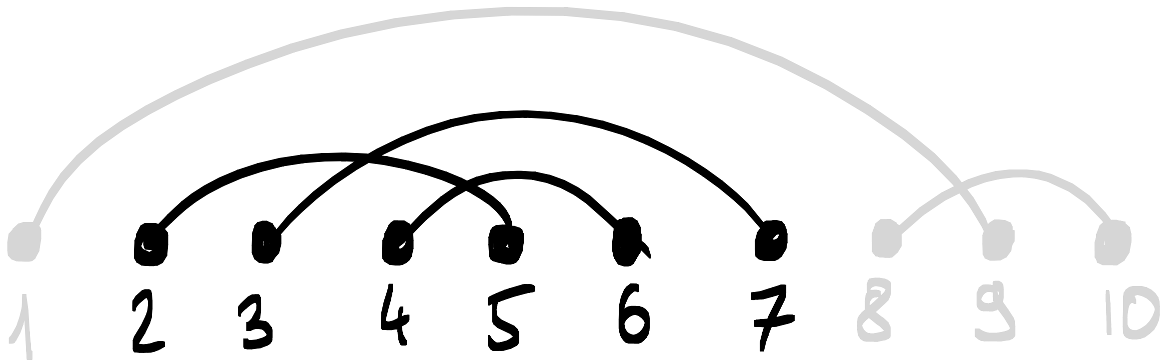

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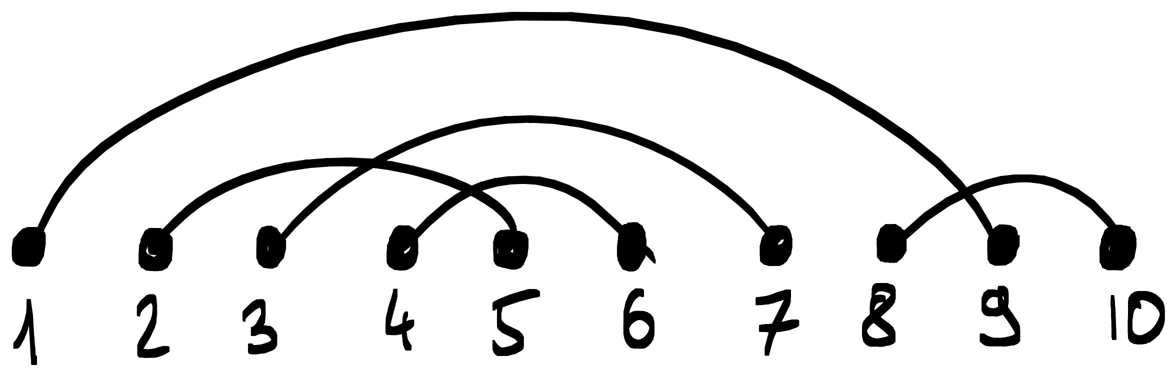

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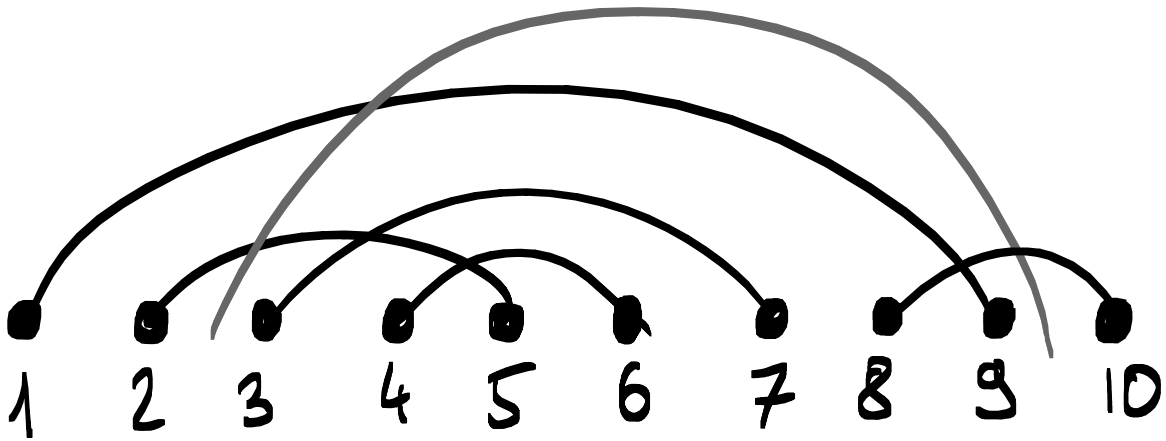

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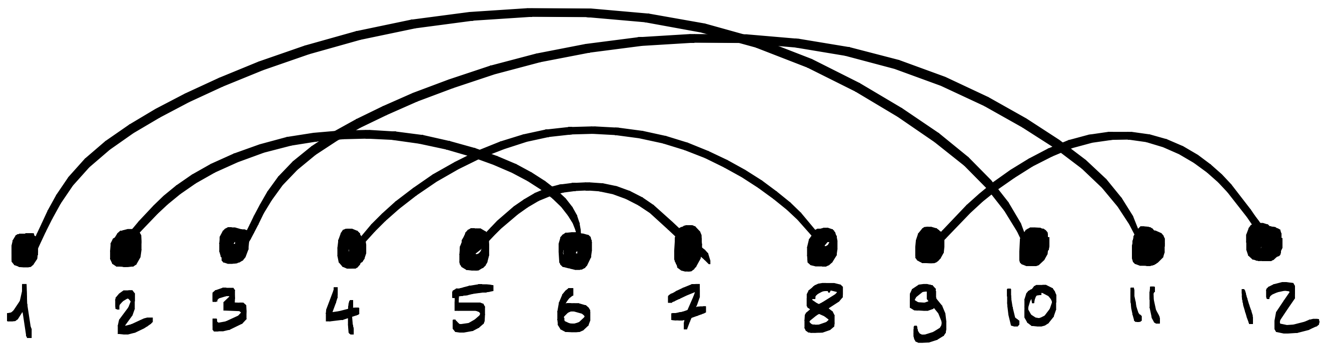

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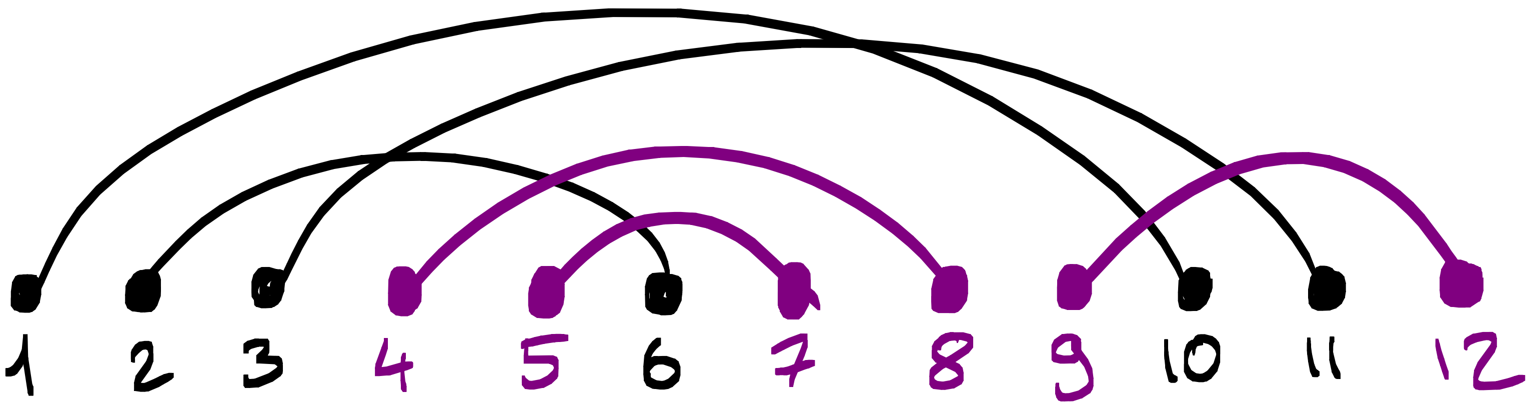
TERMINAL CHORDS



terminal chord = chord (a, b) such that for every chord (c, d) that intersects it,

$$c < a < d < b.$$

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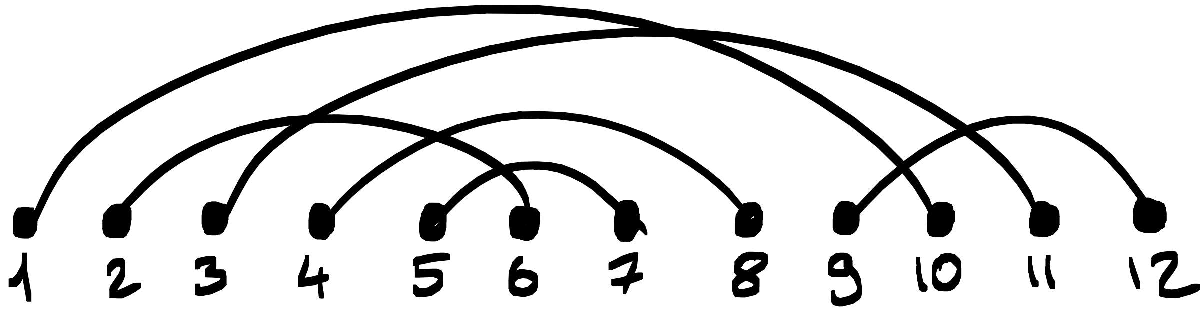
INTERSECTION ORDER



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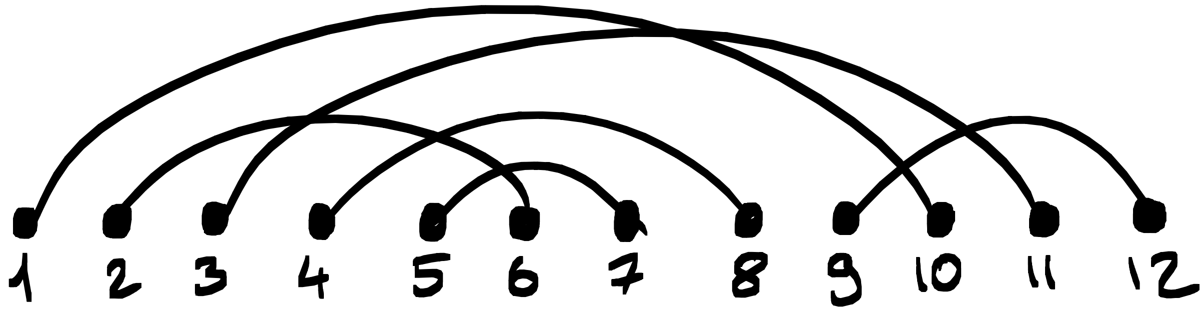

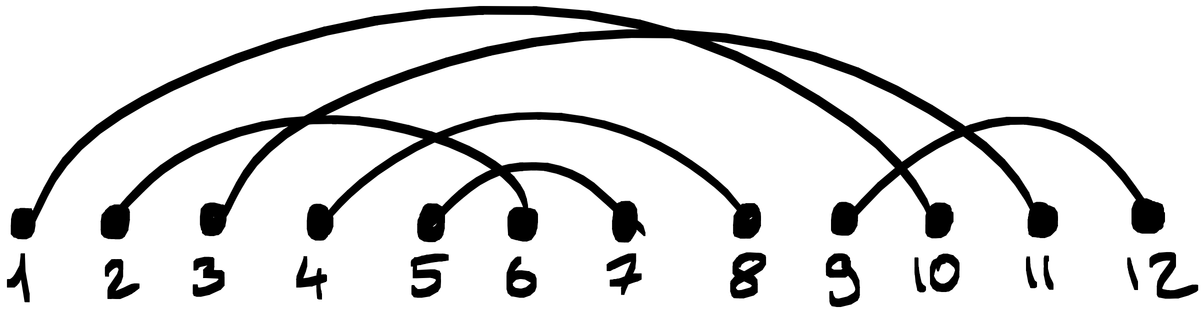

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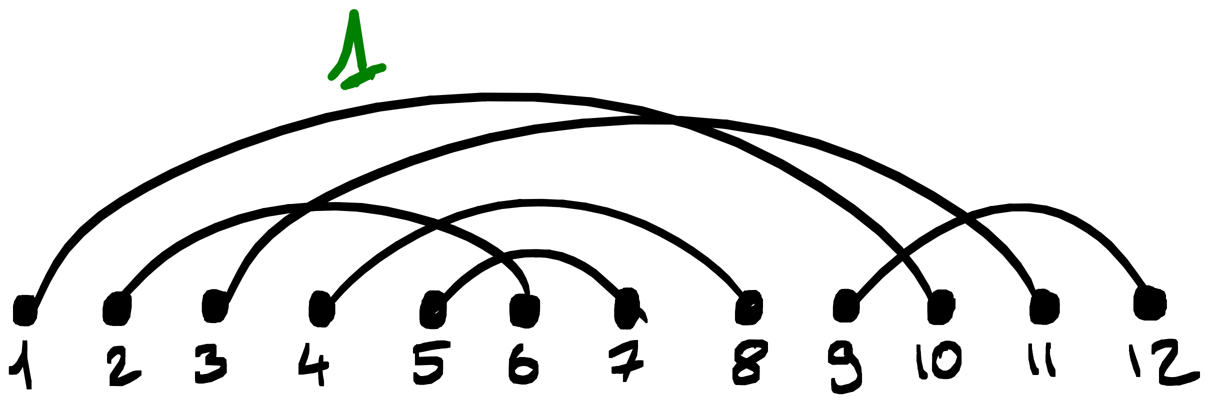

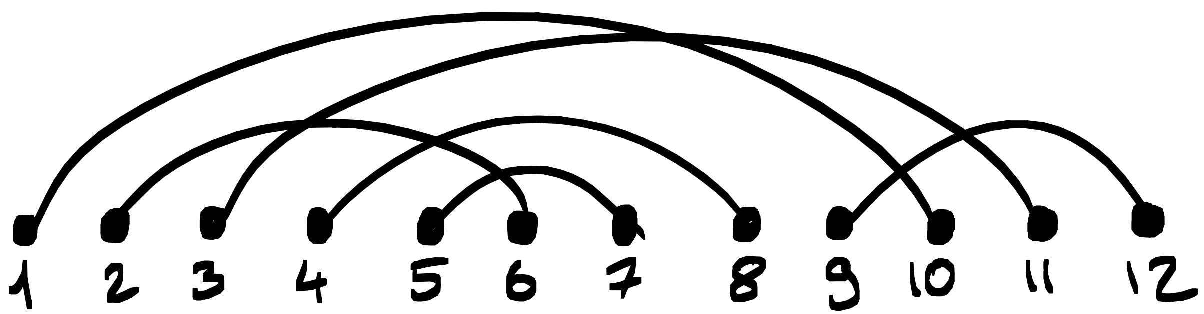

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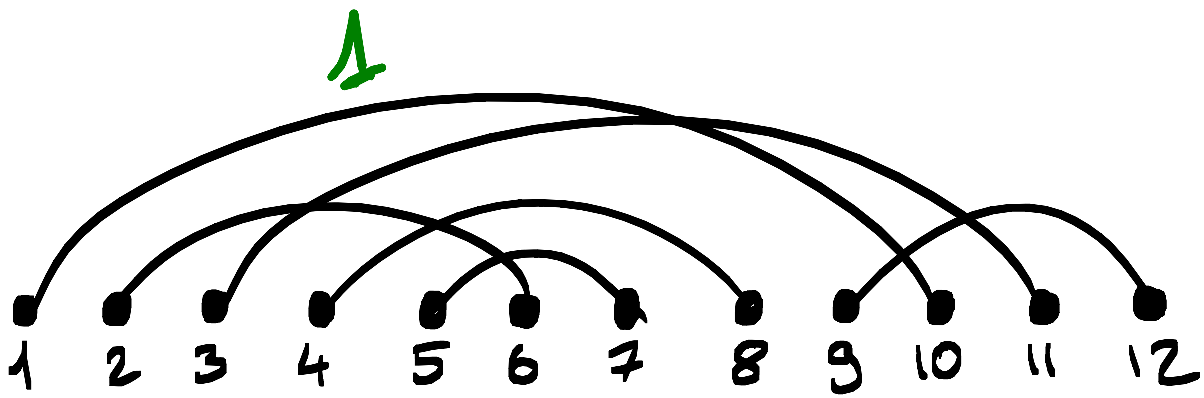

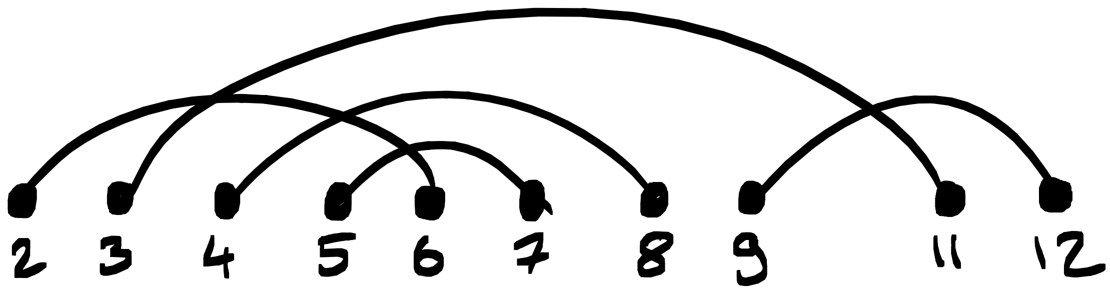

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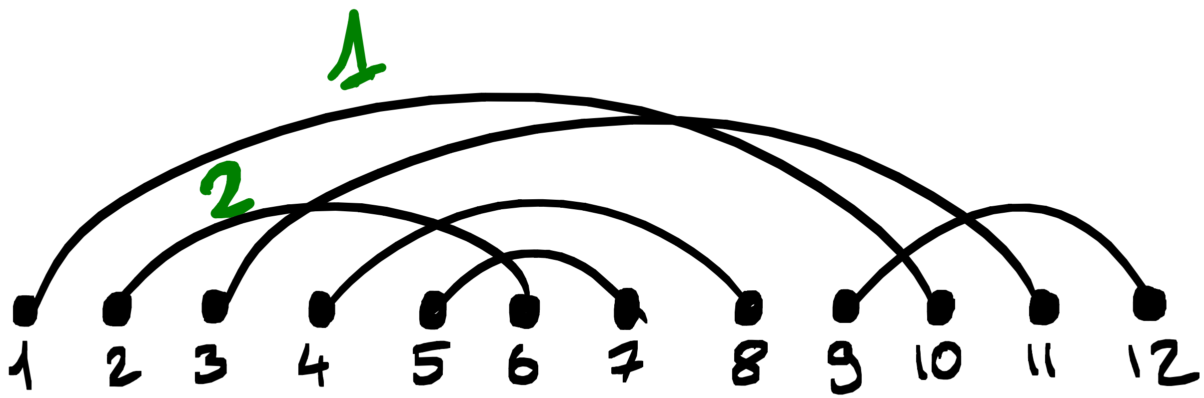

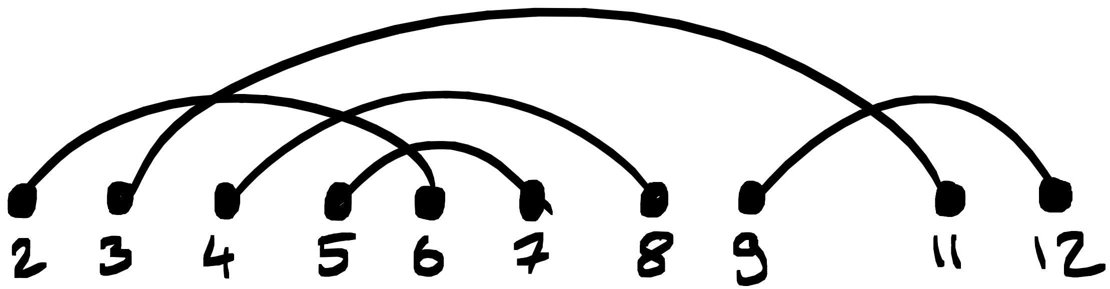

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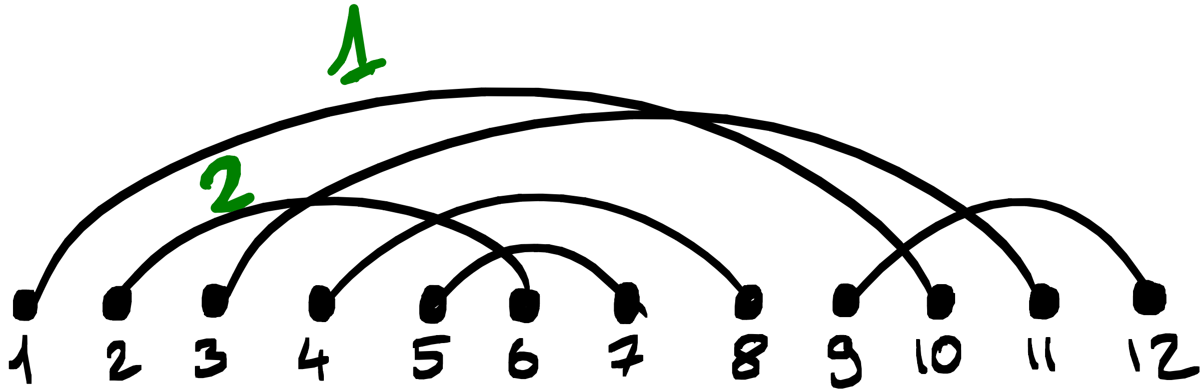

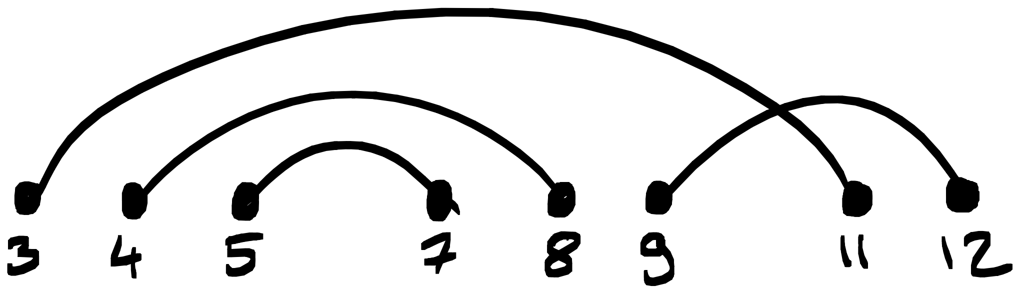

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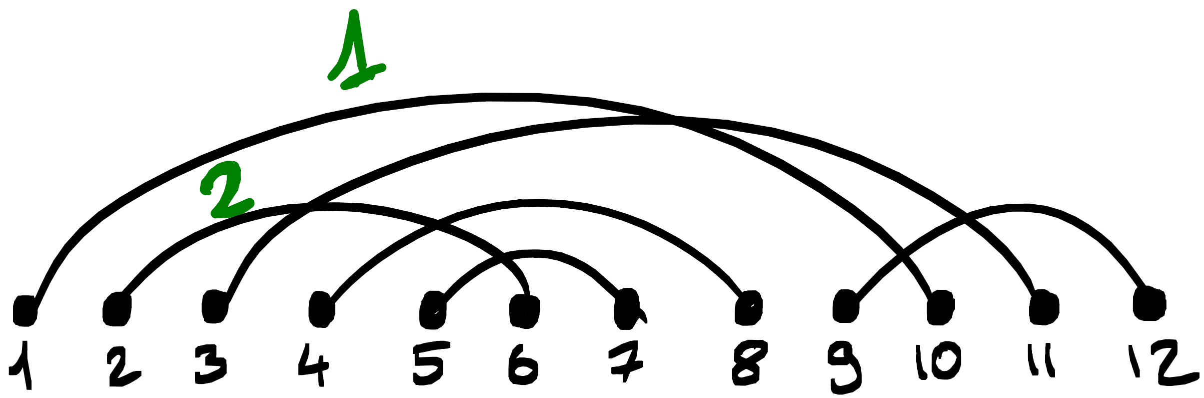

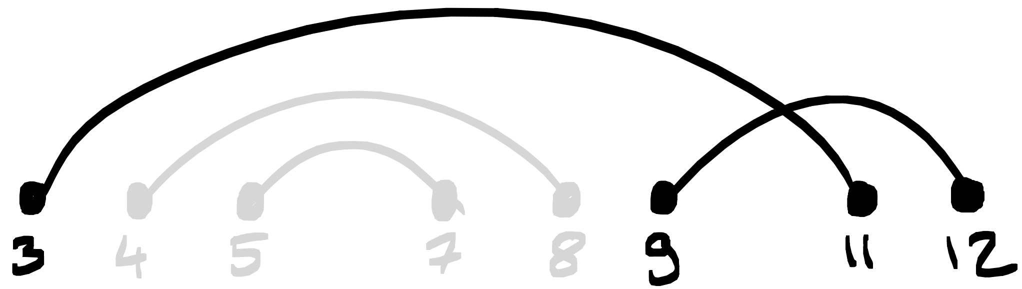

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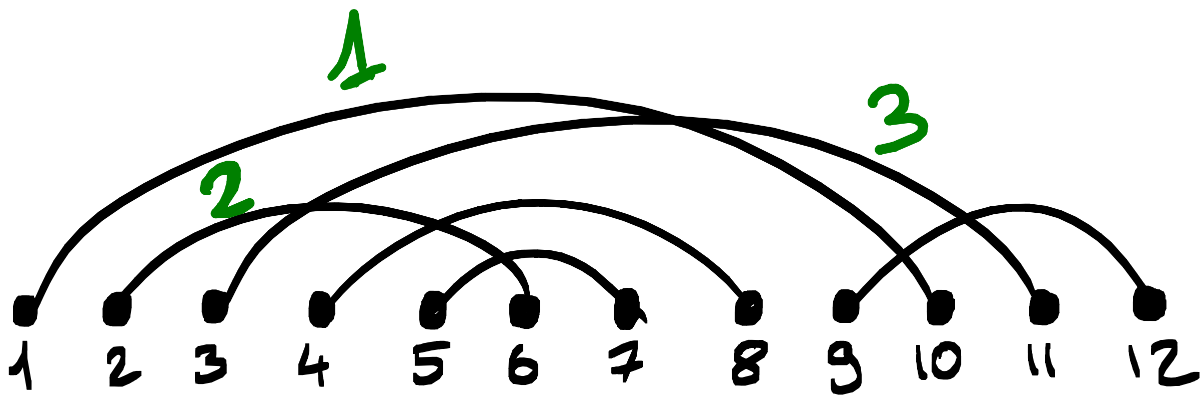

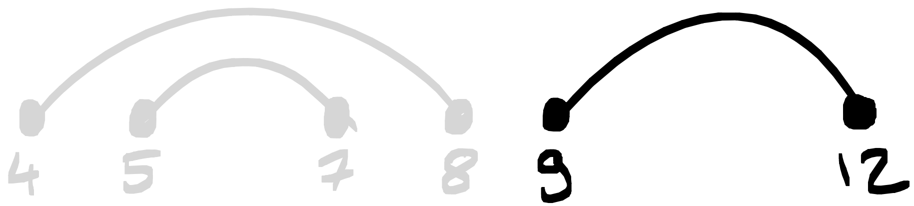

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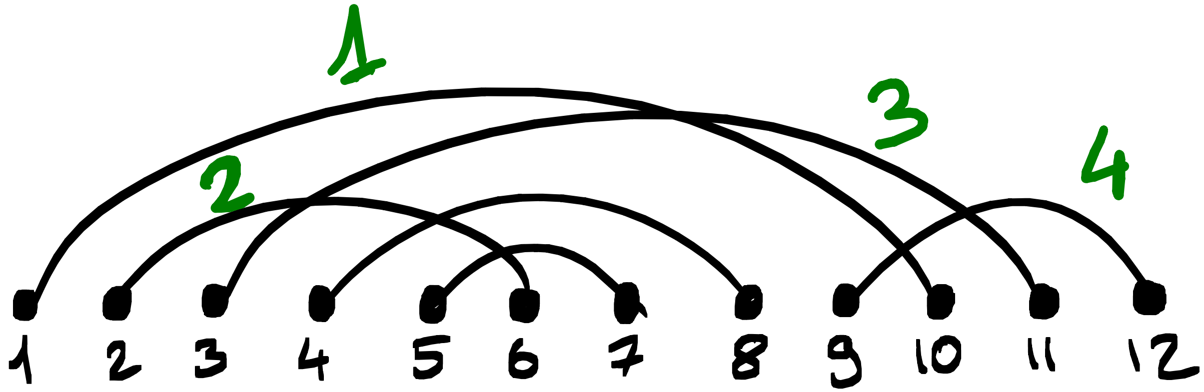

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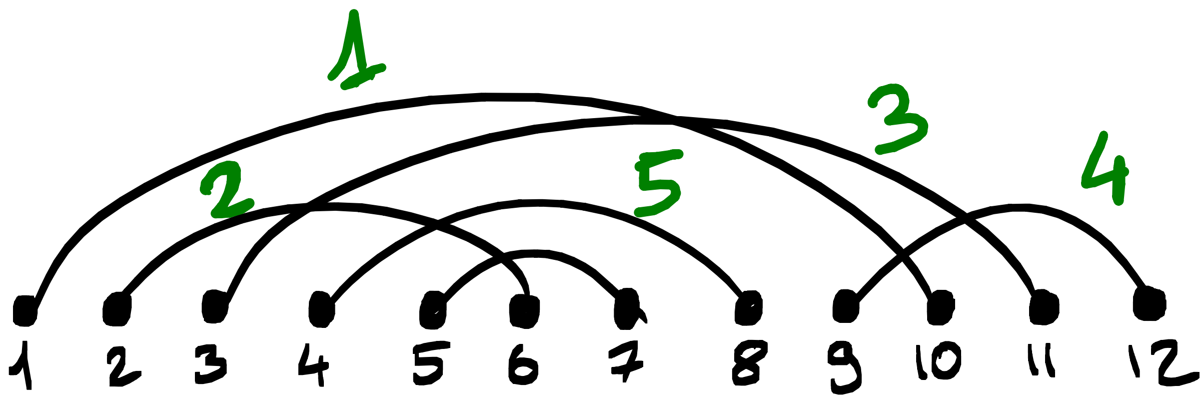

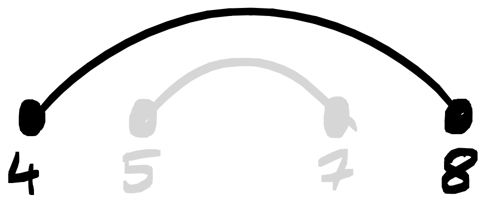

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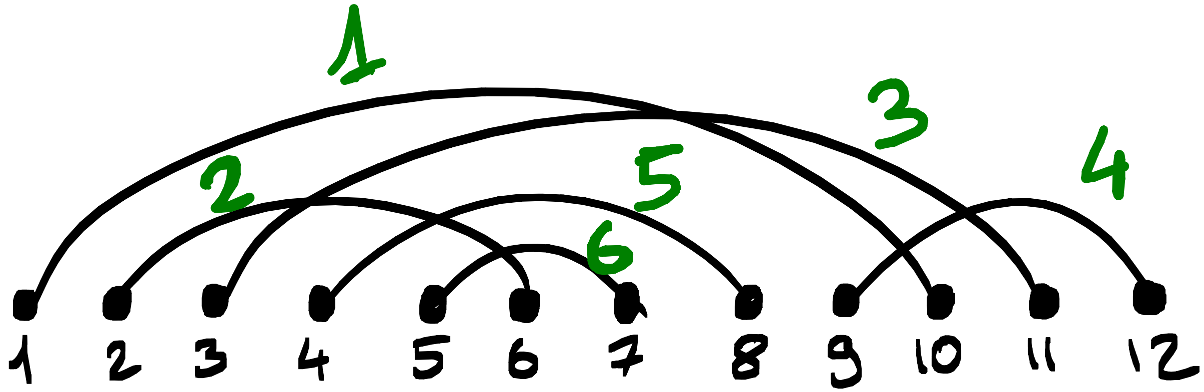

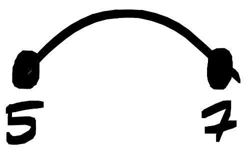

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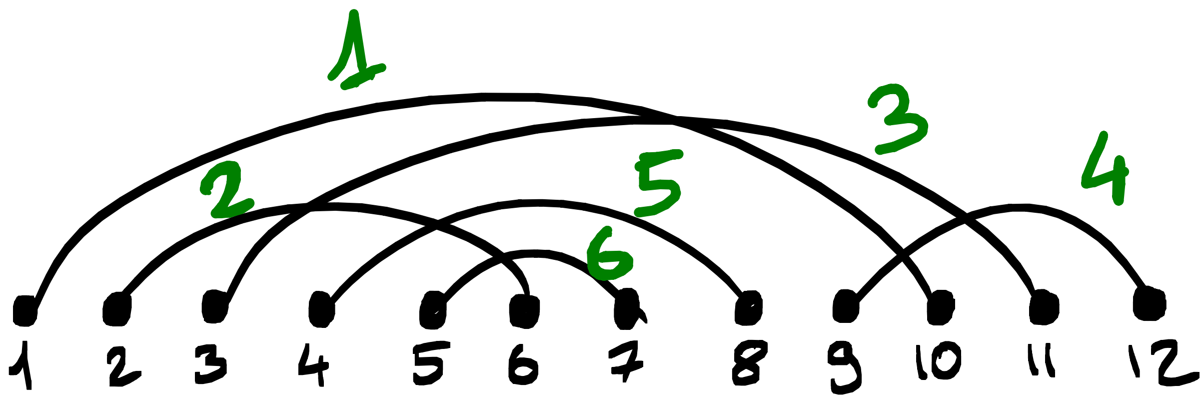

⚠ intersection order ≠ left-right order

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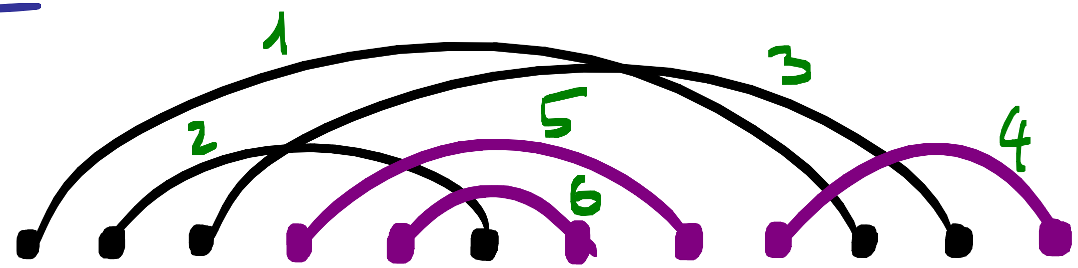
Ex:



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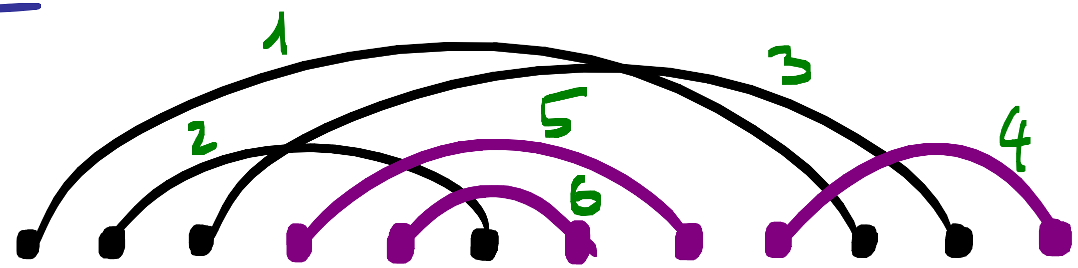

$k=3 \quad t_1=4 \quad t_2=5 \quad t_3=6$

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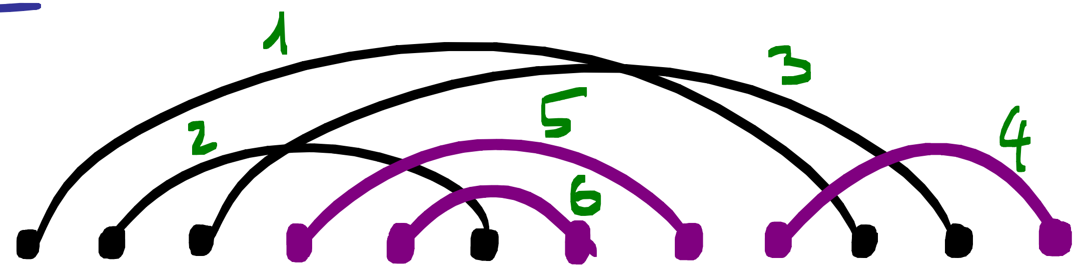

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(for $i \leq 4$)

$$\frac{L^i}{i!} x^6 \times f_0^3 \times f_{4-i} \times f_1 \times f_1$$

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QUESTIONS

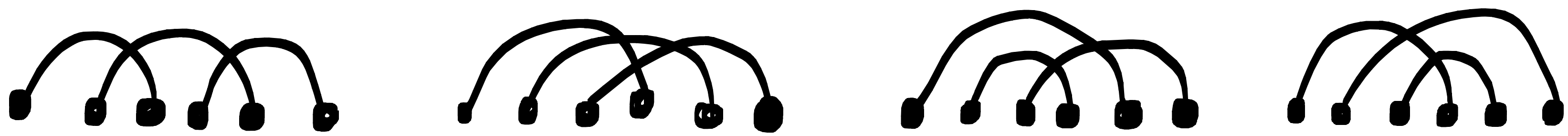
- → leading-log coefficients behaviour?
- → number of terminal chords?
- → position of the first terminal chord?
- → gaps between two terminal chords?

STEIN FORMULA

c_n = number of connected diagrams with n chords

$c_1 = 1 \quad c_2 = 1 \quad c_3 = 4 \quad c_4 = 27 \quad c_5 = 248$

For $n = 3$,



Theorem [Stein-Everett]

$$c_n = (n-1) \sum_{k=1}^{n-1} c_k \times c_{n-k}$$

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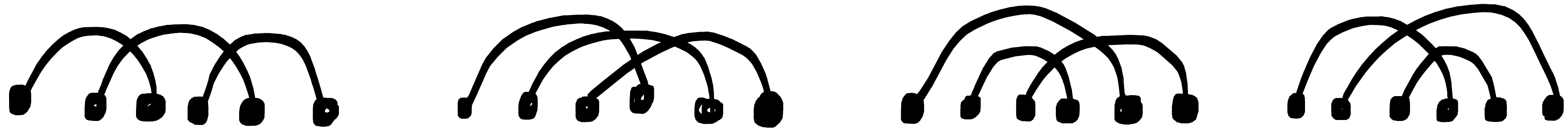

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$$c_n = (n-1) \sum_{k=1}^{n-1} c_k \times c_{n-k}$$

$$c_n \sim \frac{1}{e} \times (2n-1)!!$$

LEADING-LOG TERMS

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Coefficient of $(Lx)^i x^{|C|-i}$ for i close to $|C|$?

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Coefficient of $(Lx)^i x^{|C|-i}$ for i close to $|C|$?

- $|C| = i$: leading-log expansion [Krüger-Kreimer]
- $|C| = i+1$: next-to leading-log expansion
- $|C| = i+\ell$: next-to$^{\ell}$ leading-log expansion

LEADING-LOG TERMS

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Coefficient of $(Lx)^i x^{|C|-i}$ for i close to $|C|$?

- $|C| = i$: leading-log expansion [Krüger-Kreimer]

$\Leftrightarrow$ $t_1 = |C|$

$\Leftrightarrow$ There is only one terminal chord.

ONLY ONE TERMINAL CHORD

number of connected diagrams with n chords and only one terminal chord

$= ?$

ONLY ONE TERMINAL CHORD

number of connected diagrams with n chords and only one terminal chord

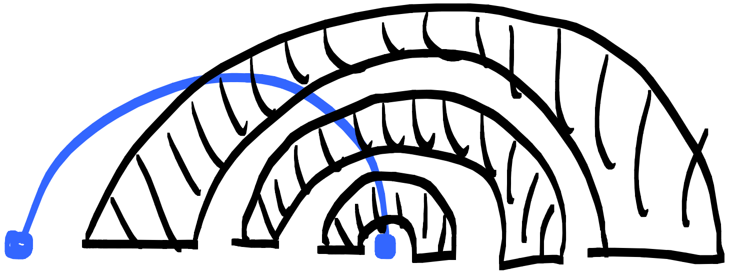
$$= (2n-3)!!$$

ONLY ONE TERMINAL CHORD

number of connected diagrams with n chords and only one terminal chord

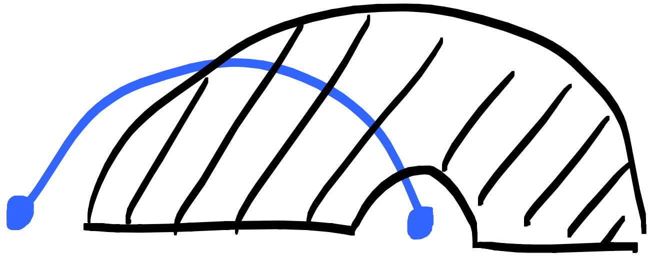
$$= (2n-3)!!$$

Proof:



impossible

One piece of size $n-1$



$2n-3$ possible locations

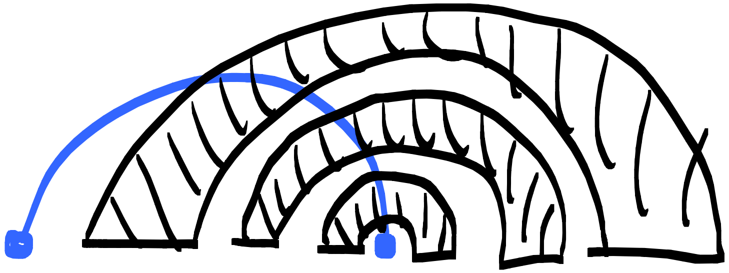
ONLY ONE TERMINAL CHORD

number of connected diagrams with n chords and only one terminal chord

$$= (2n-3)!!$$

Proof:

One piece of size $n-1$



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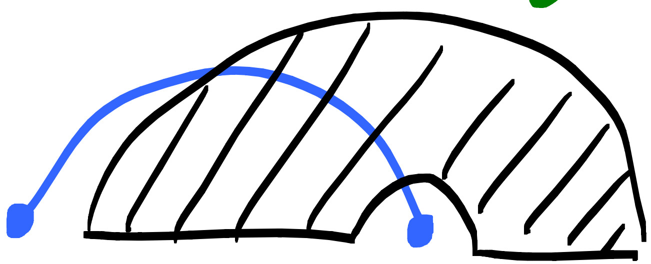

$2n-3$ possible locations

Cor: n^{th} coeff of the leading-log expansion

$$= \frac{(2n-3)!!}{n!} f_0^{\,n}$$

NEXT-TO$^{\ell}$ LEADING-LOG TERMS

→ "Similar" recursions exist for the diagrams such that $t_1 \geqslant |C| - \ell$

→ Analytic combinatorics techniques

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But how about $\beta_0^{|C|-k}\, \beta_{t_1 - i}\, \beta_{t_2 - t_1}\, \beta_{t_3 - t_2} \cdots \beta_{t_k - t_{k-1}}$?

THE LAST ℓ CHORDS ARE TERMINAL

→ "Similar" recursions exist for the diagrams such that the last ℓ chords are terminal

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Here $f_0^{|C|-k} f_{t_1-i} f_{t_2-t_1} f_{t_3-t_2} \cdots f_{t_k-t_{k-1}} = f_0^{n-\ell+1} f_1^{\ell-1}$

NEXT-TO$^{\ell}$ LEADING-LOG TERMS

Diagrams such that the last ℓ chords are terminal are dominant among the diagrams such that $t_1 \geq |C| - \ell$.

Corollary: For $\ell \geq 0$,

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Only f_0 and f_1 matter!

NUMBER OF TERMINAL CHORDS

Average number of terminal chords ?

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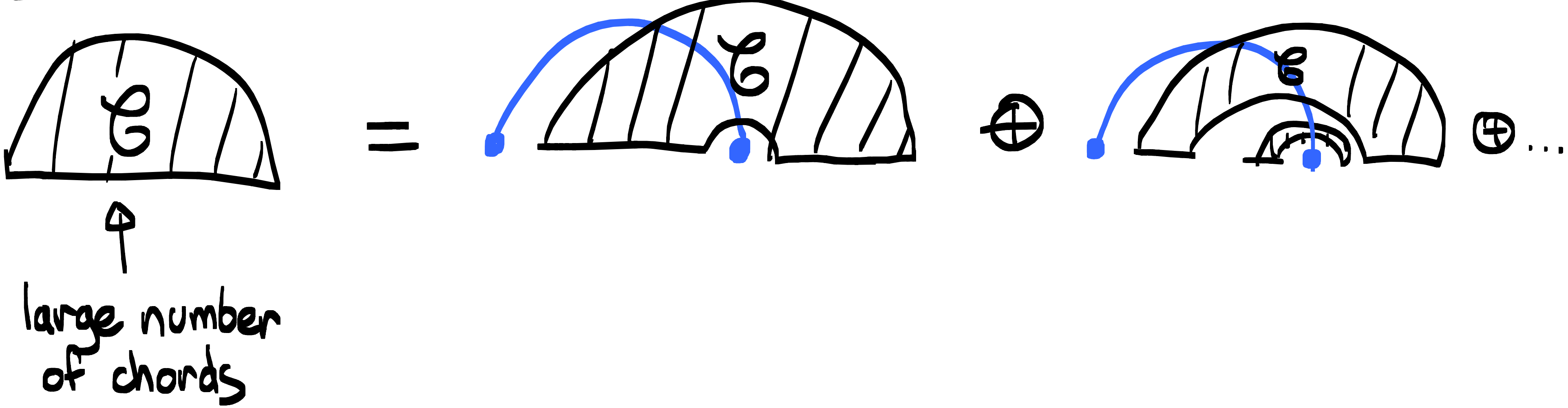
Idea:



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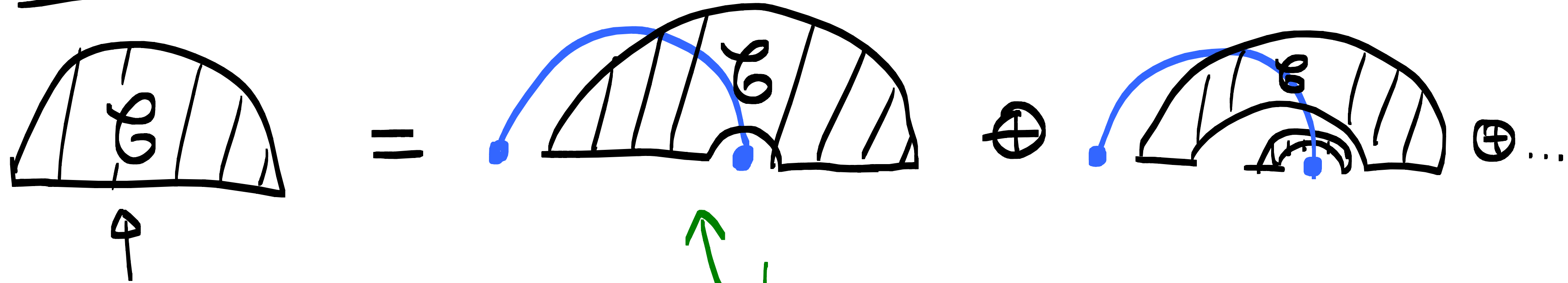

large number of chords

proba

$$= \frac{(2n-3)\, c_{n-1}}{c_n}$$

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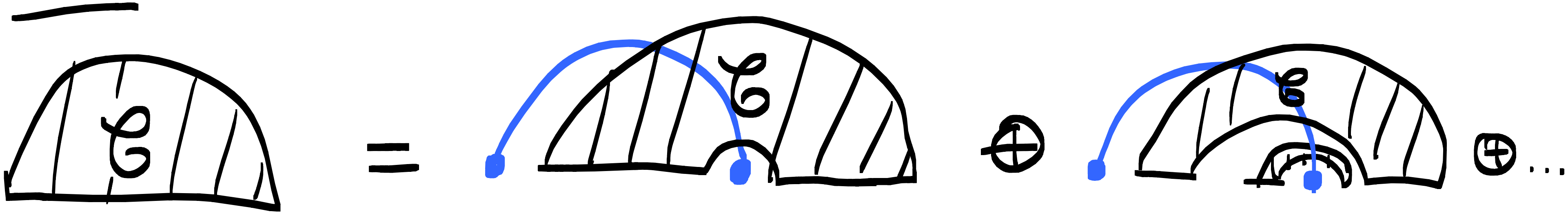

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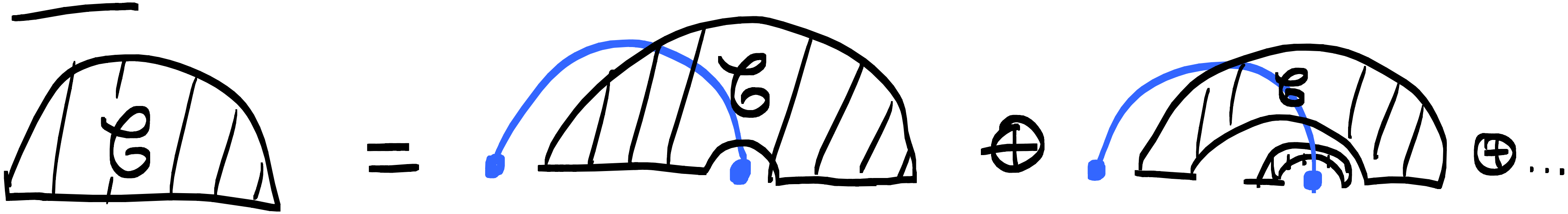

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Interesting but not sufficient...

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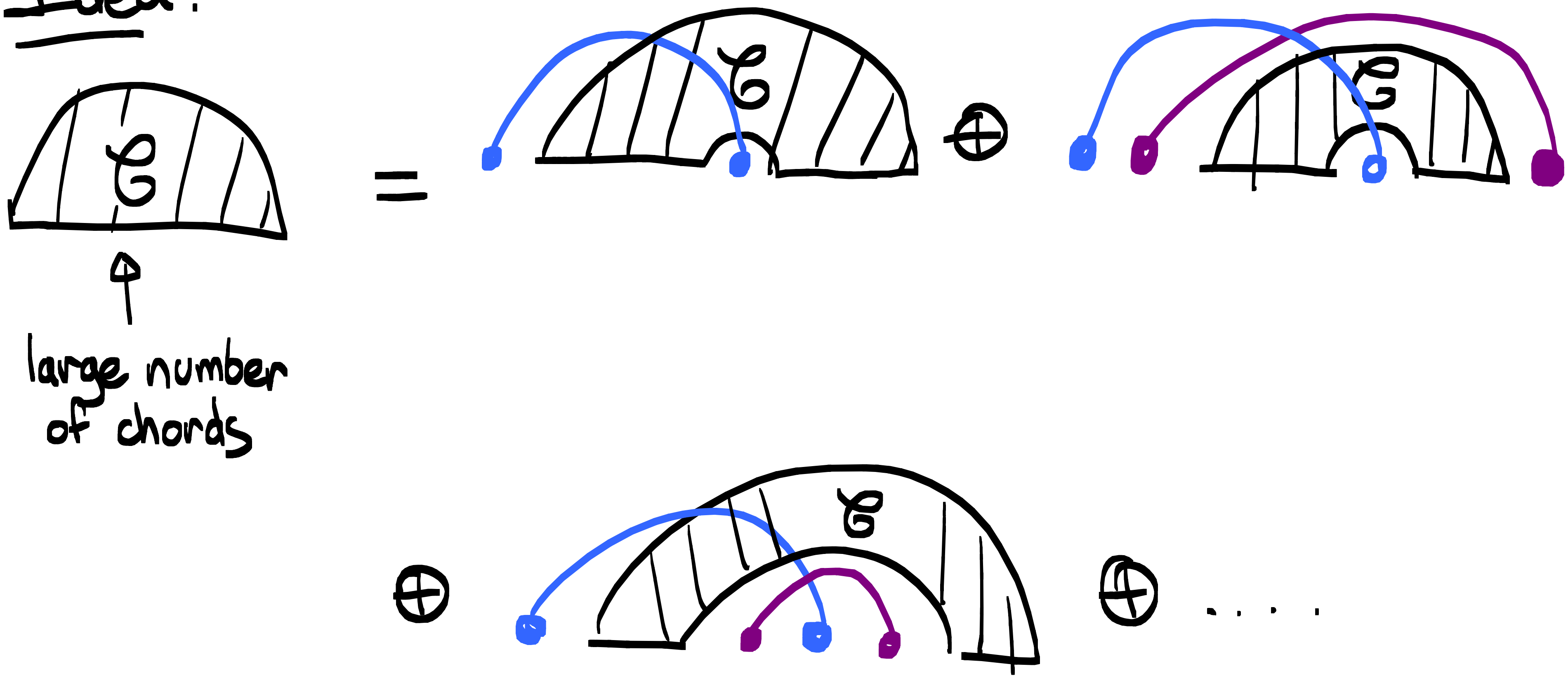

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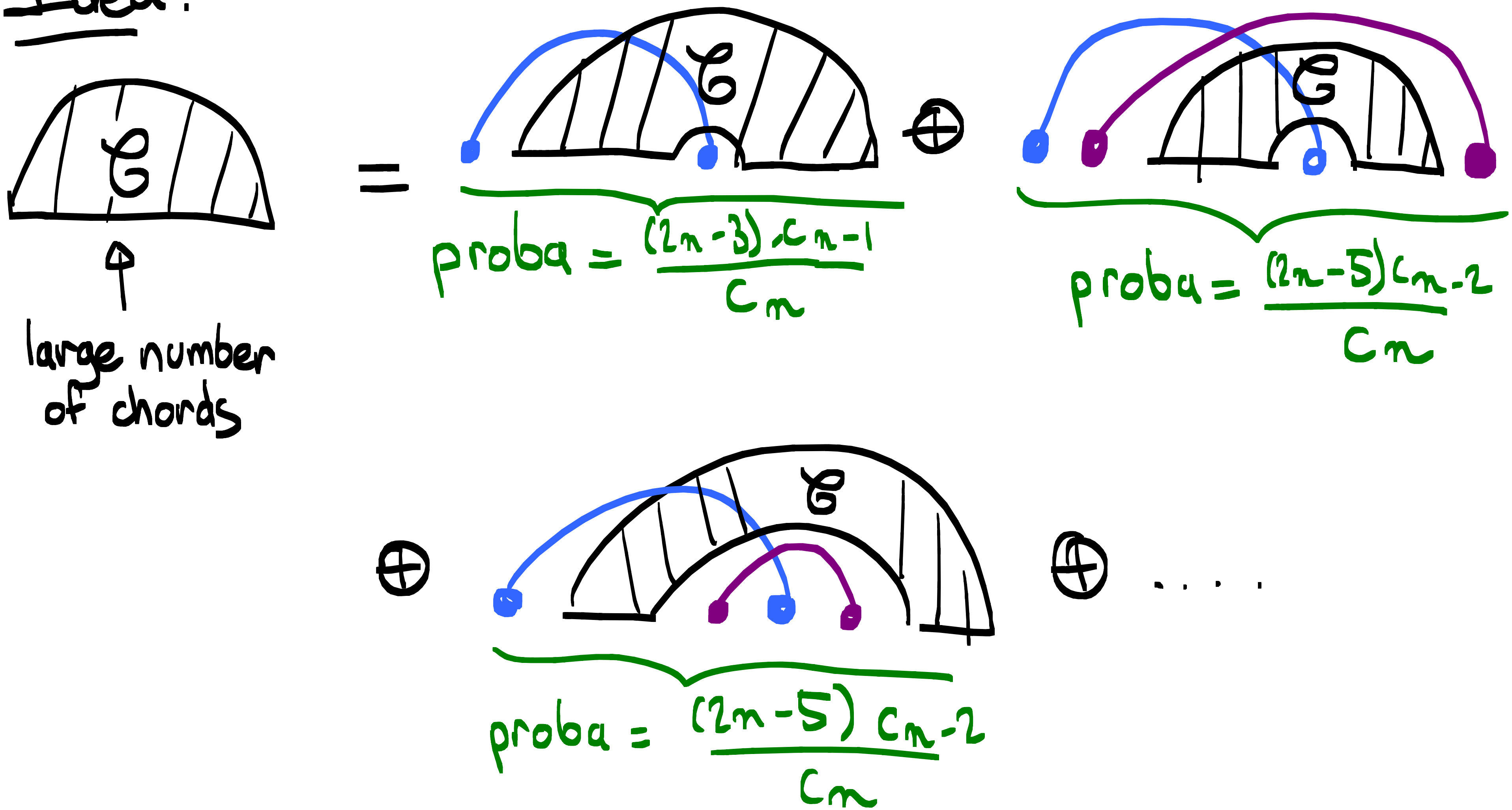

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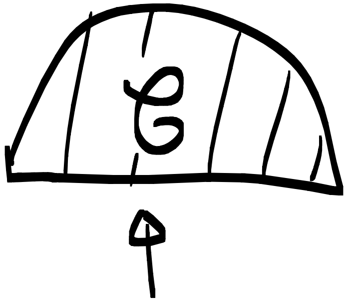

large number of chords

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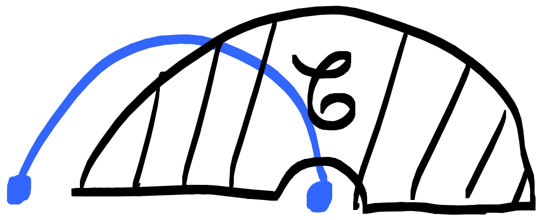
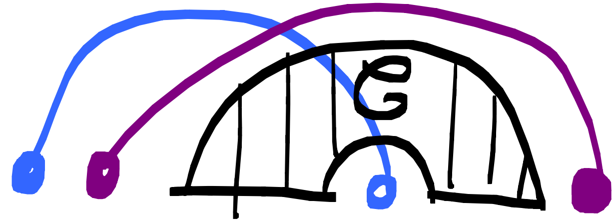


$$\text{proba} = \frac{(2n-3)\,c_{n-1}}{c_n} = 1 - \frac{1}{n} + o\left(\frac{1}{n}\right)$$

$\oplus$



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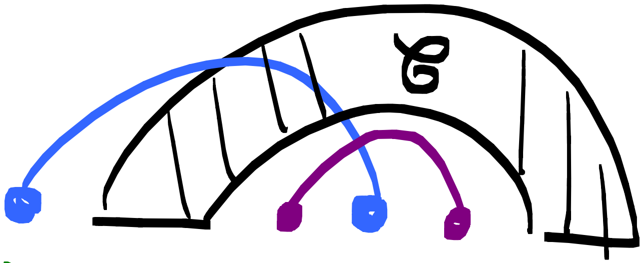

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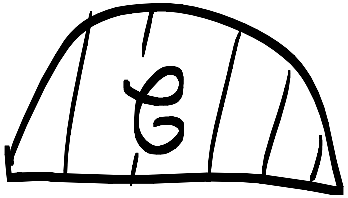

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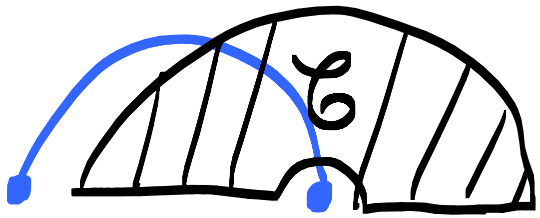

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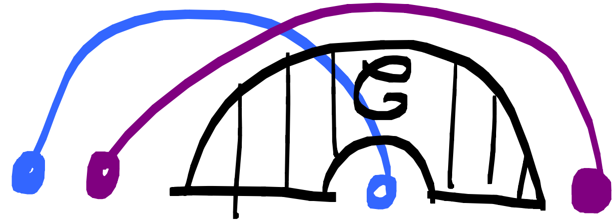

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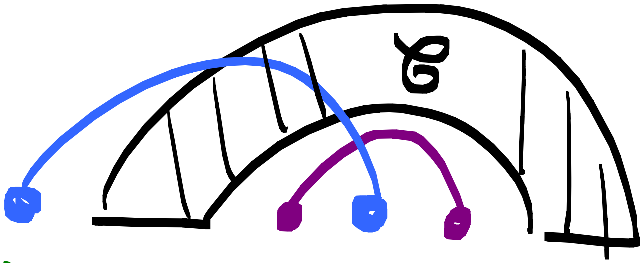

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NUMBER OF TERMINAL CHORDS

Set $p_{n,k} = \left(1-\frac{1}{n}\right) p_{n-1,k} + \frac{1}{n}\, p_{n-2,k-1}$

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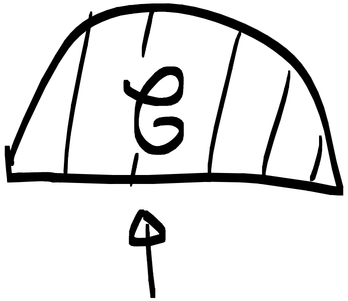

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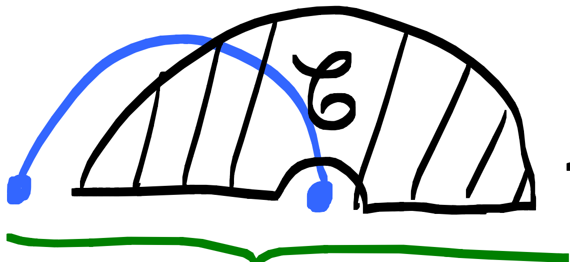

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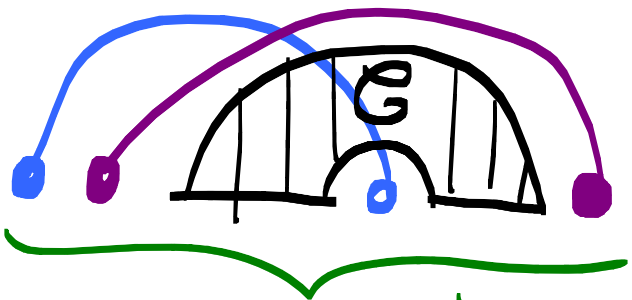

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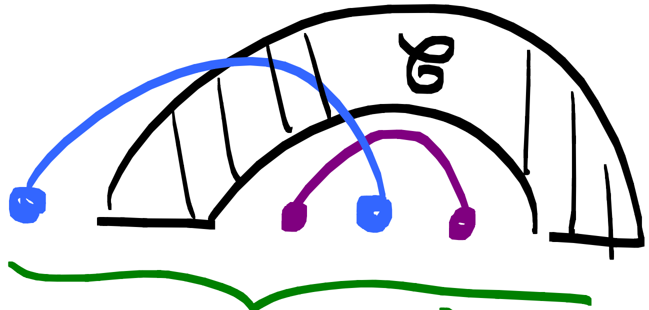

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<u>Fact 1</u>: Let X_n be the random variable such that $\mathbb{P}(X_n = k) = p_{n,k}$

$$X_n \longrightarrow \text{Gaussian law.}$$

<u>Fact 2</u>: The number of terminal chords "$\sim$" X_n

NUMBER OF TERMINAL CHORDS

Theorem : The number of terminal chords in a random connected diagram of size n asymptotically obeys to a Gaussian limit law of mean and variance $\sim \ln(n)$.

NUMBER OF CONSECUTIVE TERMINAL CHORDS

If the terminal chords are in position $t_1 < \dots < t_k$, how many j's satisfy $t_j - t_{j-1} = 1$?

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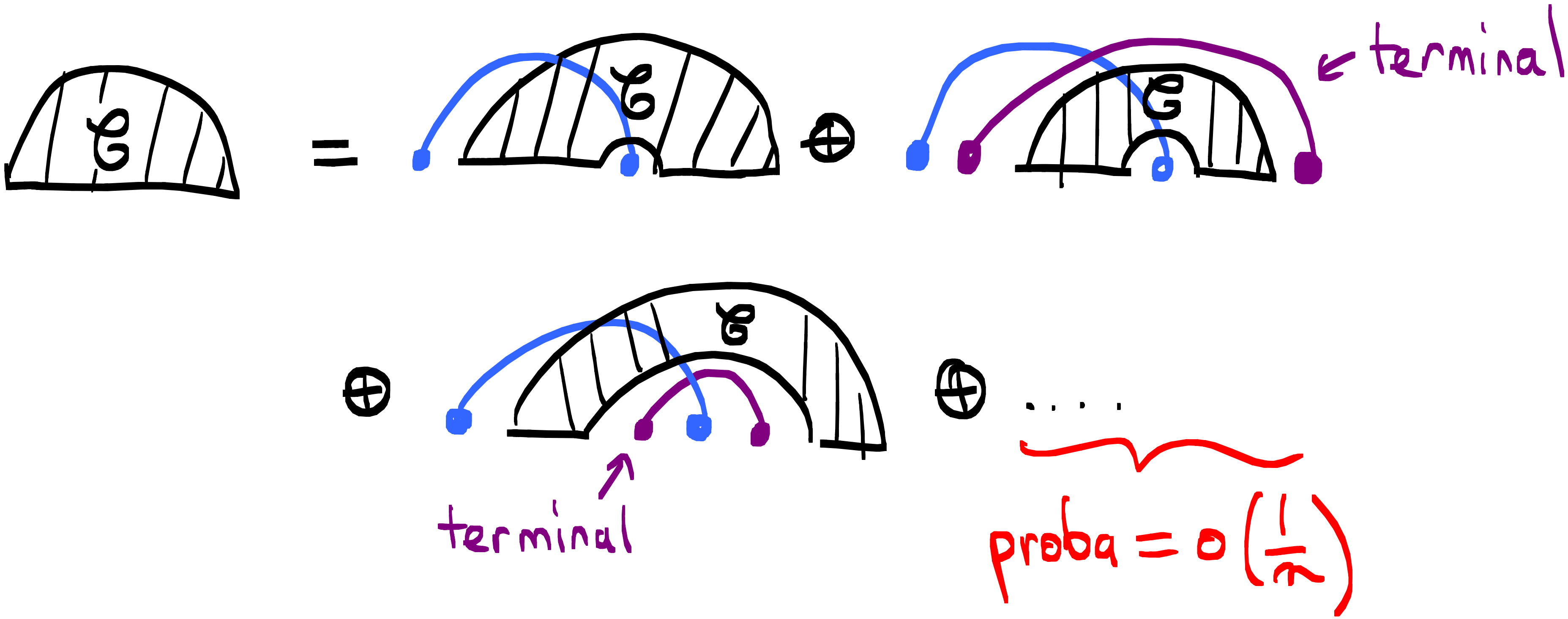

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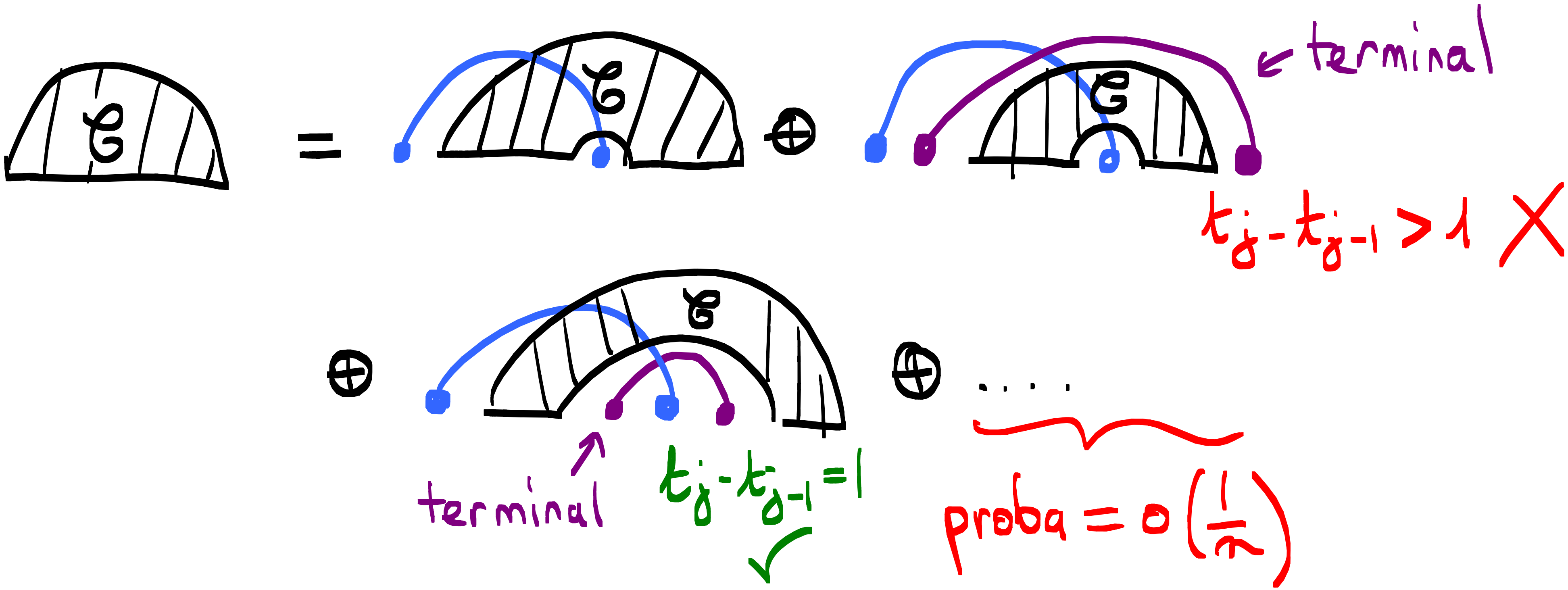

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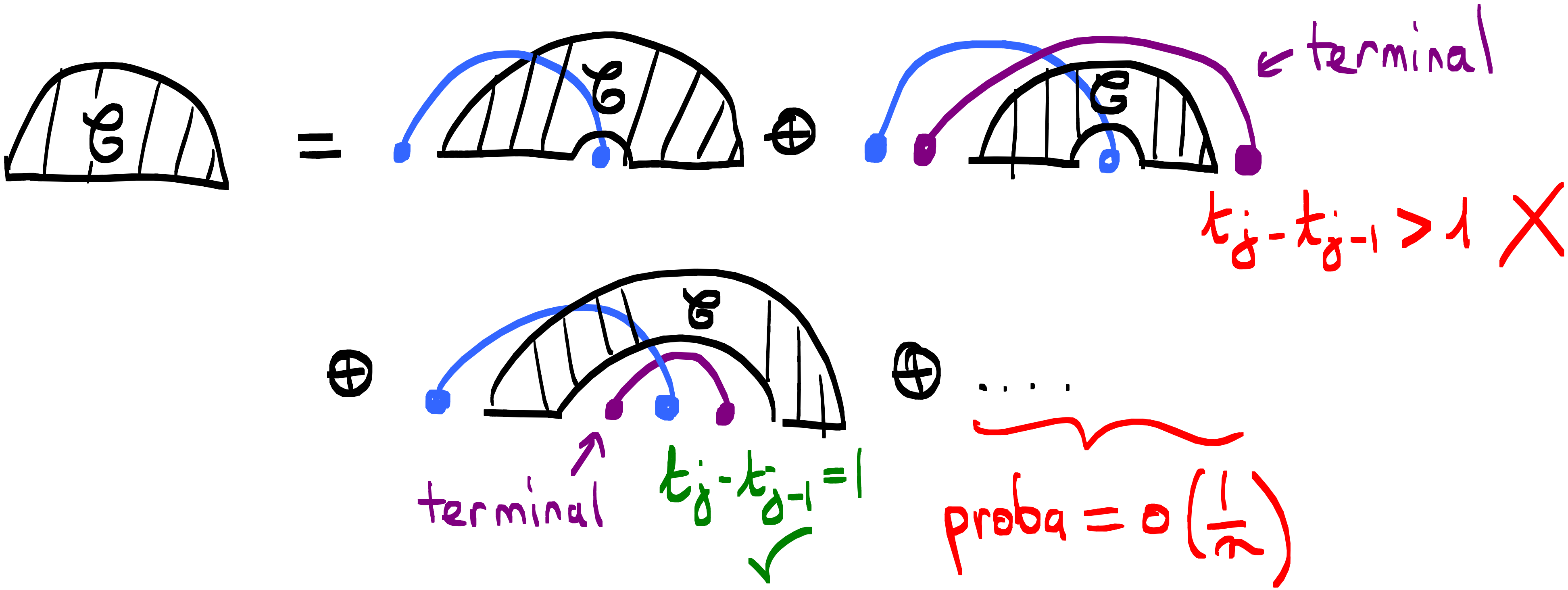

Theorem: Number of consecutive terminal chords $\longrightarrow$ Gaussian law of mean and variance $\sim \frac{\ln n}{2}$

NUMBER OF CONSECUTIVE TERMINAL CHORDS

If the terminal chords are in position $t_1 < \dots < t_k$, how many j's satisfy $t_j - t_{j-1} = 1$?

On average,

$$f_0^{|C|-k} f_{t_1 - i} f_{t_2 - t_1} f_{t_3 - t_2} \cdots f_{t_k - t_{k-1}} \sim f_0^{n - \ln n} f_{t_1 - i} f_1^{\frac{\ln n}{2}} \dots$$

→ confirms the importance of f_0 and f_1

<u>Theorem</u>: Number of consecutive terminal chords → Gaussian law of mean and variance $\sim \frac{\ln n}{2}$

POSITION OF THE FIRST TERMINAL CHORD.

t_1 = random variable returning the position of the 1st terminal chord.

Theorem: $\mathbb{E}(t_1) \sim \frac{2}{3} n$

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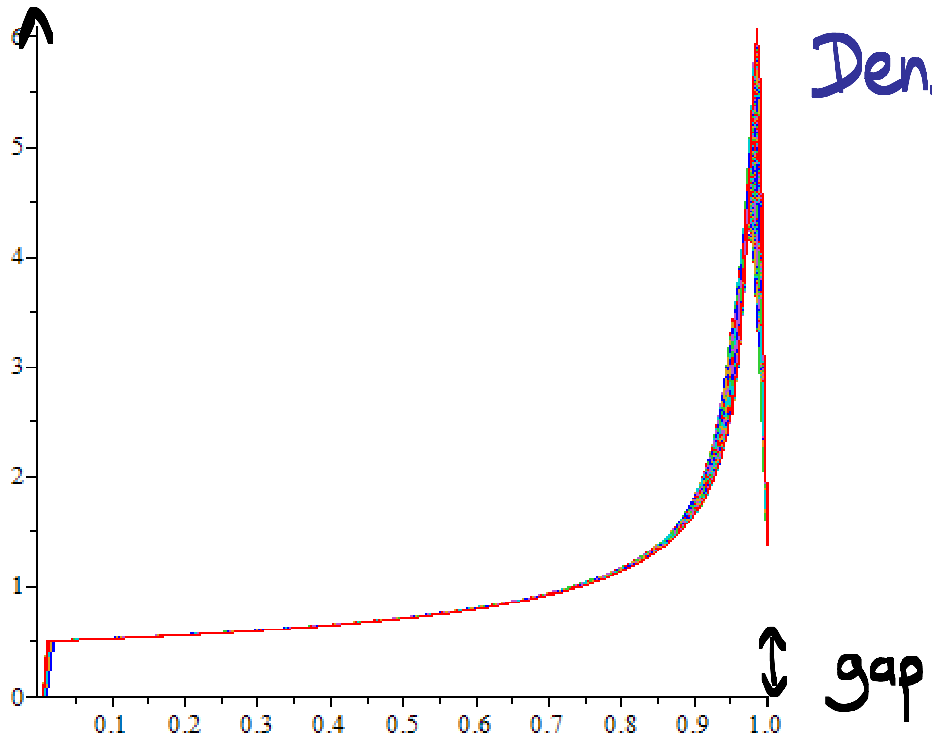
Limit law?



Density of $\frac{t_1}{n}$

CONCLUSION

→ Recovers the results of Krüger and Kreimer
  + automaticity of the method
  + asymptotic behaviour

→ New combinatorial approach

→ Extension to Hihn-Yeats's results?

THANK YOU!